Strengthened Senior High School

DepED
DEPARTMENT OF EDUCATION

Learning Activity Sheets for General Mathematics

CORE SUBJECT



Learning Activity Sheet for General Mathematics **Term 2**

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Unit 6. Piecewise Functions			
Name:		**Grade & Section:**	

Lesson 6. Illustrating a Piecewise Function in Practical Contexts	
Objectives:	At the end of the lesson, you should be able to: 1. define what a piecewise function is and explain why it is necessary for certain real-world scenarios; 2. identify the domain intervals and corresponding sub-functions in a given real-life situation; 3. identify and interpret the correct graph of a piecewise function by analyzing boundary points (open and closed circles) and interval behaviors; 4. evaluate a piecewise function $f(x)$ for specific given values of x by first determining which interval the input belongs to; and 5. formulate a piecewise function mathematical model $f(x)$ from a word problem involving fare rates, bulk purchasing, or income tax brackets.
Key Ideas and Examples:	**Piecewise Function** ***What is Piecewise Function?*** A piecewise function is a unique type of mathematical function that does not use a single, uniform formula for its entire graph. Instead, it is a "hybrid" function composed of multiple distinct equations (called ***sub-functions***), each of which applies only to a specific part of the domain (called an ***interval*** or ***sub-domain***). In other words, the rule or formula changes depending on the value of the input ***x***. ***The Anatomy of a Piecewise Function*** When written out formally, a piecewise function uses a large grouping bracket ({) to bundle all of its rules together. It looks like this: $$f(x) = \begin{cases} 2x + 1, & \text{if } x < 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$$ To read and understand this structure, think of it as a set of logical "**If-Then**" instructions: • ***The Domain Condition (The "If"):*** Your eyes must always look at the right side first. If your input number is less than 0, you must send it to the top formula. If it is greater than or equal to 0, you send it to the bottom formula. • ***The Rule (The "Then"):*** Once you've found the correct interval, you perform the math on the left side. ***Rules of Piecewise Functions*** 1. *The Intervals Cannot Overlap (Mutually Exclusive)* A fundamental rule of any mathematical function is that one input (x) can only produce exactly one output (y). Therefore, the intervals on the right side must never overlap. For instance, you will never see $x \leq 2$ and $x \geq 2$ as two separate rules because if you plugged in $x = 2$, the math system would break down trying to give you two different answers. 2. *The Graph Can Be Disconnected* Because a piecewise function is stitched together from different equations, its graph often features sudden breaks, jumps, or steps. A graph that breaks apart is called *discontinuous*, while a piecewise graph where the segments perfectly touch at the boundaries is called *continuous*. **Graph of Piecewise Functions** To graph a piecewise function accurately, follow these three core steps: *Step 1: Identify the Boundary Lines (The "Switch" Values)* Look at the inequalities on the right side of the function. These values indicate where one rule ends and another begins. It is often helpful to lightly sketch vertical dashed lines at these x-values on your coordinate plane as temporary guidelines.

Step 2: Plot Each Segment Using a Table of Values

Treat each sub-function as an independent graphing problem but *only calculate points within its allowed interval*. Always calculate the exact boundary value for the equation, even if the inequality uses a strict less-than (<) or greater-than (>) sign. This tells you exactly where the line terminates.

Step 3: Apply Open and Closed Circles at the Transitions

This is the most critical step to ensure your graph remains a valid function passing the *Vertical Line Test*:

- **Closed Circle (●):** Used when the endpoint is included in the interval. Look for the "or equal to" symbols (≤ or ≥).
- **Open Circle (○):** Used when the endpoint is *not* included. Look for strict inequalities (< or >).

Examples

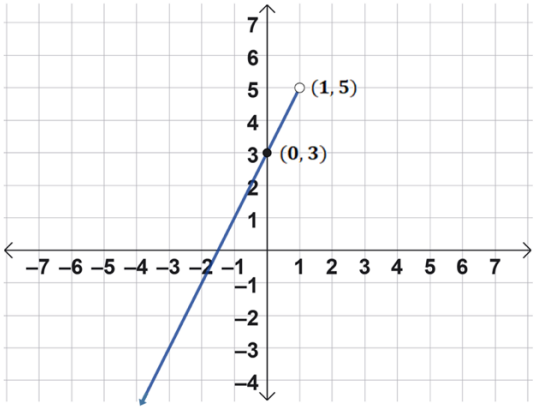
1. Graphing a Two-Part Function

Let's look at how to graph this system:

$$f(x) = \begin{cases} 2x + 3, & \text{if } x < 1 \\ -x^2 + 4, & \text{if } x \geq 1 \end{cases}$$

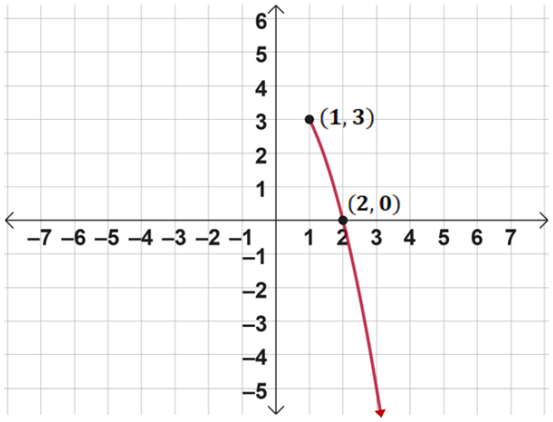
To graph the linear subfunction where $x < 1$, first identify its basic properties. The graph is a straight line with a slope of 2 and a y −intercept at $(0, 3)$. Next, find the boundary value by substituting $x = 1$ into the function. This gives a value of 5, so plot an open circle at $(1, 5)$. Use an open circle because the inequality is strictly less than.

After that, pick a few x −values smaller than 1 to find supporting points. Plugging in $x = 0$ gives $(0, 3)$, and plugging in $x = -1$ gives $(-1, 1)$. Finally, draw a straight line starting from the open circle at $(1, 5)$. Pass the line through your supporting points and extend it downward to the far left with an arrow.

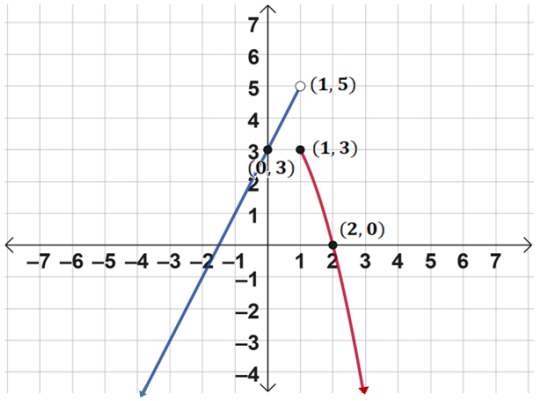


To graph the quadratic subfunction for $x \geq 1$, first identify its shape. The graph is a downward-opening parabola. Normally, its peak vertex is at $(0, 4)$. However, only draw it where $x \geq 1$. Next, find the boundary by substituting $x = 1$ into the formula. This gives a value of 3. Plot a closed circle at $(1, 3)$. Use a closed circle because the inequality is "greater than or equal to."

After that, choose x −values larger than 1 for supporting points. Plugging in $x = 2$ gives the point $(2, 0)$. Finally, draw a smooth, curved parabolic arc. Start at the closed circle $(1, 3)$. Bend the curve downward through $(2, 0)$. Extend it to the right with an arrow.



Hence, the graph of the piecewise function $f(x)$ is

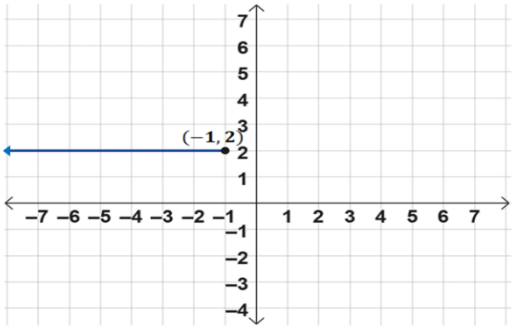


To check for a jump discontinuity, look closely at the boundary line where $x = 1$. Determine if the linear and quadratic parts connect perfectly. They do not meet because the linear graph ends at a height of $y = 5$. Meanwhile, the quadratic graph starts lower down at a height of $y = 3$. This visible gap or vertical break means the two lines fail to touch. Therefore, this graph represents a *discontinuous* piecewise function.

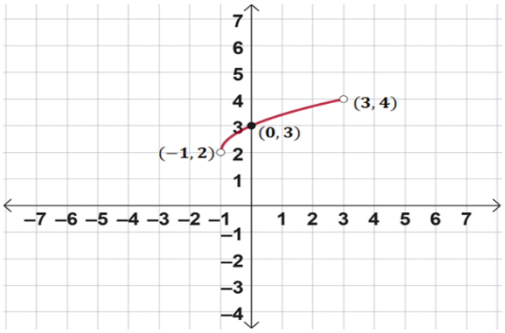
2. Graphing a Three-Part Function
Let's look at how to graph this system:

$$f(x) = \begin{cases} \mathbf{2}, & \textbf{if } \boldsymbol{x \leq -1} \\ \boldsymbol{\sqrt{x+1}+2}, & \textbf{if } \boldsymbol{-1 < x < 3} \\ \boldsymbol{|x-5|+2}, & \textbf{if } \boldsymbol{x \geq 3} \end{cases}$$

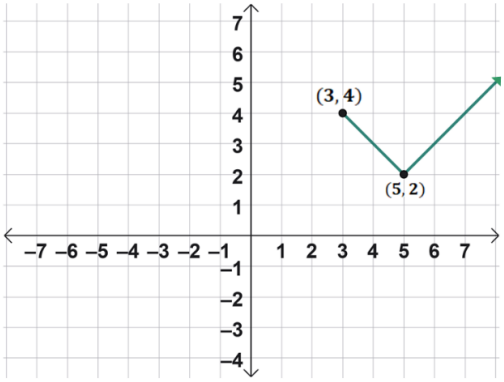
To graph this three-part piecewise function accurately, it is best to break the system down and visualize each subfunction independently along with its critical boundary points. Succeedingly, we will look at the separate components labeled as graphs a, b, and c to see how each individual equation behaves before they are combined. First, graph a represents the constant subfunction $f(x) = 2$ for the interval $x \leq -1$, which ends at the solid critical point $(-1, 2)$. Next, graph b illustrates the square root subfunction $f(x) = \sqrt{x+1} + 2$ for the interval $-1 < x < 3$, bounded between the open critical points $(-1, 2)$ and $(3, 4)$ while passing through the y-intercept at $(0, 3)$. Finally, graph c displays the absolute value subfunction $f(x) = |x - 5| + 2$ for the interval $x \geq 3$, which begins at the solid critical point $(3, 4)$ and dips down to its vertex at $(5, 2)$.



a

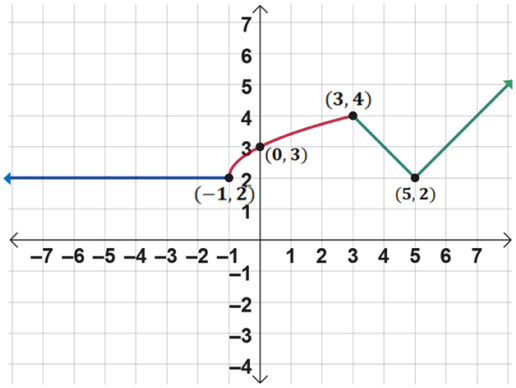


b



c

Layering the subfunctions onto one plane creates graph *d*. This graph proves that $f(x)$ is a *continuous piecewise function*. It is continuous because it forms an unbroken line with no gaps or jumps. The solid endpoints from the outer pieces perfectly plug the open holes of the middle piece. You can trace the entire path without lifting your pencil.



d

Representing Real-Life Situation using Piecewise Functions

To represent any real-world scenario into a mathematically sound piecewise function, we use a systematic 4-step process:

1. Define the Variables (Identify your input $'x'$ and output $'y'$)
2. Pinpoint the Breakpoints (Identify exactly where the rules change)
3. Construct the Domain Intervals (Build the mathematical inequalities)
4. Formulate the Sub-functions (Write the algebraic equations)

Example 1

A local tricycle driver charges a fixed minimum fare of ₱15.00 for the first 4 km or less. For any distance beyond 4 km, the passenger pays the base ₱15.00 plus an additional ₱2.00 per kilometer for the extra distance traveled. Represent the total tricycle fare for the distance traveled using the function $f(x)$, where x is the number of distances traveled.

Step 1: Define the Variables

Input (x): Distance traveled (in kilometers)
Output $(f(x))$: Total tricycle fare (in Pesos)

Step 2: Pinpoint the Breakpoints

The payment structure shifts gears exactly at $x = 4$.

Step 3: Construct the Domain Intervals

Interval 1 (Minimum Fare): Distance from 0 up to 4 kilometers is $0 < x \leq 4$
Interval 2 (Over-distance Fare): Distance strictly greater than 4 kilometers is $x > 4$

Step 4: Formulate the Sub-functions

For $0 < x \leq 4$: The rate is fixed at ₱15.00
For $x > 4$: The base fare (₱15.00) plus ₱2.00 for every kilometer exceeding 4, which is written as $(x - 4)$,

$$15 + 2(x - 4) = 15 + 2x - 8$$
$$= \mathbf{2x + 7}$$

The Piecewise Model

$$f(x) = \begin{cases} 15, & \text{if } 0 < x \leq 4 \\ 15 + 2(x - 4), & \text{if } x > 4 \end{cases}$$

or

$$f(x) = \begin{cases} 15, & \text{if } 0 < x \leq 4 \\ 2x + 7, & \text{if } x > 4 \end{cases}$$

Example 2.

For employees receiving monthly compensation, the Bureau of Internal Revenue determines tax withholding based on specific gross monthly earnings brackets. Monthly earnings up to ₱20,833.33 are completely tax-free. Earnings above ₱20,833.33 and up to ₱33,333.33 are taxed at a rate of 15% of the amount exceeding ₱20,833.33. For individuals earning above ₱33,333.33 and up to ₱66,666.67, the tax consists of a flat, fixed amount of ₱1,875.00 plus an additional 20% of the amount exceeding ₱33,333.33. Represent the total monthly income tax $f(x)$, as a piecewise function of an employee's gross monthly earnings x.

Step 1: Define the Variables

Input (x): Employee's gross monthly earnings (in Pesos)
Output $(f(x))$: Total monthly income tax withholding (in Pesos)

Step 2: Pinpoint the Breakpoints

The tax rules switch gears at two explicit income thresholds:
First switch: $x = 20{,}833.33$
Second switch: $x = 33{,}333.33$
Upper limit mentioned: $x = 66{,}666.67$

Step 3: Construct the Domain Intervals

Interval 1: Gross earnings up to ₱20,833.33 is

$$0 \leq x \leq 20{,}833.33$$

Interval 2: Earnings above ₱20,833.33 up to ₱33,333.33 is

$$20{,}833.33 < x \leq 33{,}333.33$$

Interval 3: Earnings above ₱33,333.33 up to ₱66,666.67 is

$$33{,}333.33 < x \leq 66{,}666.67$$

Step 4: Formulate the Sub-functions

For $0 \leq x \leq 20{,}833.33$: Tax is 0.

For $20{,}833.33 < x \leq 33{,}333.33$: Taxed at 15% (0.15) of the amount exceeding ₱20,833.33:

$$0.15(x - 20{,}833.33) = 0.15x - 3{,}125$$

For $33{,}333.33 < x \leq 66{,}666.67$: A flat fixed amount of ₱1,875.00 plus 20% (0.20) of the amount exceeding ₱33,333.33:

$$1{,}875 + 0.20(x - 33{,}333.33) = 1{,}875 + 0.20x - 6{,}666.67$$
$$= 0.20x - 4{,}791.67$$

The Piecewise Model

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq 20{,}833.33 \\ 0.15(x - 20{,}833.33), & \text{if} 20{,}833.33 < x \leq 33{,}333.33 \\ 1{,}875 + 0.20(x - 33{,}333.33), & \text{if } 33{,}333.33 < x \leq 66{,}666.67 \end{cases}$$

or

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq 20{,}833.33 \\ 0.15x - 3{,}125, & \text{if} 20{,}833.33 < x \leq 33{,}333.33 \\ 10.20x - 4{,}791.67, & \text{if } 33{,}333.33 < x \leq 66{,}666.67 \end{cases}$$

Example 3

A custom textile manufacturer offers bulk discounts for printing institutional shirts. The cost per shirt is ₱250 for orders of 1 to 20 shirts, ₱220 for 21 to 50 shirts, ₱190 for 51 to 100 shirts, and ₱150 for orders of more than 100 shirts. Express the total cost $f(x)$, as a piecewise function of the number of shirts ordered x.

Step 1: Define the Variables

Input (x): The number of shirts ordered

Output ($f(x)$): The total purchase cost of the entire order (in Pesos)

Step 2: Pinpoint the Breakpoints

The wholesale unit pricing drops at three explicit milestone adjustments:

First switch: After 20 shirts

Second switch: After 50 shirts

Third switch: After 100 shirts

Step 3: Construct the Domain Intervals

Interval 1: Orders from 1 to 20 shirts is $1 \leq x \leq 20$

Interval 2: Orders from 21 to 50 shirts is $21 \leq x \leq 50$

Interval 3: Orders from 51 to 100 shirts $51 \leq x \leq 100$

Interval 4: Orders of more than 100 shirts is $x > 100$

Step 4: Formulate the Sub-functions

For $1 \leq x \leq 20$, cost is $250x$.

For $21 \leq x \leq 50$, cost is $220x$.

For $51 \leq x \leq 100$, cost is $190x$.

For $x > 100$, cost is $150x$.

The Piecewise Model

$$f(x) = \begin{cases} 250x, & \text{if } 1 \leq x \leq 20 \\ 220x, & \text{if } 21 \leq x \leq 50 \\ 190x, & \text{if } 51 \leq x \leq 100 \\ 150x, & \text{if } x > 100 \end{cases}$$

Evaluating Piecewise Functions

When we evaluate a standard algebraic function like $f(x) = 3x + 1$, the process is straightforward: no matter what number we choose for x, we always plug it into that single expression.

However, evaluating a piecewise function requires a bit of detective work first. Because a piecewise function is made up of different sub-functions, you cannot just plug your input value into any formula you see. You must first look at the **domain restrictions** (the "if" conditions on the right side) to determine which specific sub-function rules that territory.

Examples.

1. Given the piecewise function $f(x)$:

$$f(x) = \begin{cases} 2x + 7, & \text{if } x < 0 \\ \sqrt[3]{x} + 4, & \text{if } x \geq 0 \end{cases}$$

a. evaluate $f(-5)$.

Solutions:

$$f(x) = 2x + 7$$
$$f(-5) = 2(-5) + 7$$
$$= -10 + 7$$
$$= -3$$

b. evaluate $f(8)$.

Solutions:

$$f(x) = \sqrt[3]{x} + 4$$
$$f(8) = \sqrt[3]{8} + 4$$
$$= 2 + 4$$
$$= 6$$

2. Given the piecewise function $g(x)$:

$$g(x) = \begin{cases} |x + 3|, & \text{if } x \leq -1 \\ x^2 - 4x + 1, & \text{if } -1 < x < 4 \\ \sqrt{x - 4} + 2, & \text{if } x \geq 4 \end{cases}$$

a. evaluate $g(3)$.

Solutions:

$$g(x) = x^2 - 4x + 1$$
$$g(3) = (3)^2 - 4(3) + 1$$
$$= 9 - 12 + 1$$
$$= -2$$

b. evaluate $g(4)$.

Solutions:

$$g(x) = \sqrt{x - 4} + 2$$
$$g(4) = \sqrt{4 - 4} + 2$$
$$= \sqrt{0} + 2$$
$$= 0 + 2$$
$$= 2$$

c. evaluate $g(-6)$.

Solutions:

$$g(x) = |x + 3|$$
$$g(-6) = |-6 + 3|$$
$$= |-3|$$
$$= 3$$

	3. Given the piecewise function $h(x)$: $$h(x) = \begin{cases} -5, & \text{if } x \leq -3 \\ 3x+1, & \text{if } -3 < x < 1 \\ \sqrt{x-1}-2, & \text{if } 1 \leq x \leq 5 \\ (x-5)^2+2, & \text{if } x > 5 \end{cases}$$ a. evaluate $h(-10)$. Solutions: $h(x) = -5$ $h(-10) = -5$ b. evaluate $h(0)$. Solutions: $h(x) = 3x + 1$ $h(0) = 3(0) + 1$ $= 0 + 1$ $= 1$ c. evaluate $h(5)$. Solutions: $h(x) = \sqrt{x-1} - 2$ $h(5) = \sqrt{5-1} - 2$ $= \sqrt{4} - 2$ $= 2 - 2$ $= 0$ d. evaluate $h(7)$. Solutions: $h(x) = (x-5)^2 + 2$ $h(7) = (7-5)^2 + 2$ $= (2)^2 + 2$ $= 4 + 2$ $= 6$
Other Resources: *(Digital Modules and Textbook, Video Lessons)*	Brian McLogan. (2025, March 03). *Understanding Piecewise Functions: Why They Confuse You.* https://www.youtube.com/watch?v=xvEprqBoqXo STEM Teacher PH. (2020, October 02). *Word Problem: Piecewise Functions - General Mathematics.* https://www.youtube.com/watch?v=fej6u3Ps1cw WOW MATH. (2020, June 09). *Piecewise Functions || Grade 11 General Mathematics Q1.* https://www.youtube.com/watch?v=l3RSLL4BMe8

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen and paper laptop
III. Instructions:	A. *Evaluate the given piecewise functions for the specified values of x.* $$\boldsymbol{f(x) = \begin{cases} \lvert x+3\rvert - 2, & \textbf{if } x < 1 \\ x^2 - 4x + 5, & \textbf{if } x \geq 1 \end{cases}}$$ $$\boldsymbol{g(x) = \begin{cases} 5, & \textbf{if } x \leq -2 \\ \sqrt[3]{x} + 4, & \textbf{if } -2 < x < 3 \\ \sqrt{x+1} + 1, & \textbf{if } x \geq 3 \end{cases}}$$ $$\boldsymbol{h(x) = \begin{cases} \lvert x-2\rvert, & \textbf{if } x \leq -4 \\ \sqrt[3]{x+1}, & \textbf{if } -4 < x < 0 \\ 7, & \textbf{if } 0 \leq x \leq 3 \\ (x-3)^2 + 2, & \textbf{if } x > 3 \end{cases}}$$

1. $f(-5)$
2. $f(1)$
3. $f(3)$
4. $g(-8)$
5. $g(0)$
6. $g(8)$
7. $h(-6)$
8. $h(-1)$
9. $h(3)$
10. $h(5)$

B. Read each real-world scenario carefully and write the complete piecewise function that models each situation.

1. A modern jeepney charges a base passenger fare of ₱13 for the first 4 kilometers traveled. For distances beyond 4 kilometers, passengers pay the base fare plus an additional ₱1.80 for every extra kilometer (or fraction thereof) exceeding the initial $4\,km$. Write a piecewise function $f(x)$ representing the total fare in pesos for a distance of x kilometers.
2. A screen-printing shop charges ₱200 per shirt for custom orders of up to 25 shirts. To encourage larger orders, the shop applies a discounted bulk rate of ₱160 per shirt for any order exceeding 25 shirts. Express the total order cost $C(x)$ in pesos as a piecewise function of the number of shirts ordered, x.
3. A commercial parking garage downtown sets its pricing based on duration. Parking for up to 2 hours costs a flat rate of ₱50. For stays longer than 2 hours and up to 6 hours, the rate increases to ₱50 plus ₱25 per hour for every hour past the first 2 hours. For long-term parking exceeding 6 hours up to 24 hours, a daily flat fee of ₱200 is charged. Model the total parking cost $P(x)$ in pesos as a piecewise function of time x in hours.
4. A local municipal water district calculates monthly residential bills based on consumption Tier:
   Tier A: 0 to 10 cubic meters (m^3) are billed at a flat minimum fee of ₱180.
   Tier B: Consumption above $10\,m^3$ up to $20\,m^3$ incurs the ₱180 base fee plus ₱22 for each cubic meter exceeding $10\,m^3$.
   Tier C: Consumption above $20\,m^3$ up to $50\,m^3$ costs ₱400 (the base cost for the first $20\,m^3$) plus ₱35 for each cubic meter exceeding $20\,m^3$.
   Write a piecewise function $W(x)$ representing the total monthly water bill in pesos for x cubic meters of water consumed.
5. A local city tax office assesses an annual municipal business tax on small businesses based on their gross annual sales or receipts using four progressive brackets:
   Bracket 1: Gross receipts from ₱0 up to ₱100,000 pay a flat tax fee of ₱0.
   Bracket 2: Gross receipts above ₱100,000 up to ₱300,000 are taxed at a flat rate of 0.5% of the amount exceeding ₱100,000.
   Bracket 3: Gross receipts above ₱300,000 up to ₱600,000 incur a base tax of ₱1,000 plus 1% of the amount exceeding ₱300,000.
   Bracket 4: Gross receipts above ₱600,000 are charged a base tax of ₱4,000 plus 1.5% of the amount exceeding ₱600,000.
   Construct a piecewise function $T(x)$ that computes the local business tax in pesos for a business earning a gross annual revenue of x pesos.

B. Graph each piecewise function on a Cartesian plane. State whether each graph is Continuous or Discontinuous.

1. $\boldsymbol{f(x) = \begin{cases} x + 3, & \text{if } x < 1 \\ x^2 + 3, & \text{if } x \geq 1 \end{cases}}$

2. $\boldsymbol{f(x) = \begin{cases} -x + 2, & \text{if } x \leq 2 \\ |x - 4| - 1, & \text{if } x > 2 \end{cases}}$

3. $\boldsymbol{f(x) = \begin{cases} |x + 2|, & \text{if } x < -1 \\ 1, & \text{if } -1 \leq x \leq 3 \\ \sqrt{x - 3} + 1, & \text{if } x > 3 \end{cases}}$

4. $f(x) = \begin{cases} (x+2)^2 - 1, & \text{if } x \leq 0 \\ 3, & \text{if } 0 < x < 4 \\ \sqrt{x} + 2, & \text{if } x \geq 4 \end{cases}$

5. $f(x) = \begin{cases} -x - 1, & \text{if } x < -2 \\ |x|, & \text{if } -2 \leq x \leq 1 \\ 1, & \text{if } 1 < x < 3 \\ \sqrt[3]{x-3} + 1, & \text{if } x \geq 3 \end{cases}$

C. Read the passage below carefully. Answer the 10 questions that follow by choosing the correct letter.

The Lifeline Electricity Rates

In the Philippines, electric distribution utilities implement a specialized tiered billing mechanism known as the Lifeline Rate Structure. Mandated by law to cushion low-income households from rising energy costs, this policy provides heavily discounted power rates to marginal consumers who consume small amounts of electricity each month.

Under this system, power charges do not follow a simple, uniform multiplication rule. Instead, the electricity bill behaves as a segmented pricing model. For instance, a household consuming between 0 kWh and 20 kWh per month receives a steep 100% discount on their generation charge - paying only a flat baseline administrative fee. Once electricity consumption crosses the threshold into the 21 kWh to 50 kWh bracket, the discount scales down to 50%. If consumption increases further to the 51 kWh to 70 kWh tier, the discount drops to 35%. For any consumption exceeding 100 kWh, no discount applies, and the standard, full rate per kilowatt-hour is charged.

The transition values such as 20 kWh, 50 kWh, and 100 kWh, act as critical demarcation points. At these exact values, the rules governing how a bill is calculated shift abruptly. Economists and policy analysts rely on mathematical functions to model these systems because a single standard formula cannot capture a rate structure where the applicable rule instantly mutates based on consumer behavior. By using mathematical conditions to partition the domain of kilowatt-hour consumption into non-overlapping intervals, utilities ensure that every single kilowatt-hour used is assigned to one, and only one, billing tier.

1. As used in paragraph 3 of the text, what does the phrase *"demarcation points"* most nearly mean?
   A. boundary markers that indicate where one rule ends and another begins
   B. the final date on which an electric bill must be paid
   C. errors in electrical meters that cause incorrect billing
   D. the highest amount of electricity a household is legally allowed to consume
2. In the sentence *"a single standard formula cannot capture a rate structure where the applicable rule instantly mutates based on consumer behavior,"* which word is a synonym for mutates?
   A. expire
   B. stabilize
   C. transforms
   D. multiplies
3. According to the text, what discount percentage on the generation charge is granted to a household that consumes between 21 kWh and 50 kWh in a month?
   A. 100%
   B. 50%
   C. 35%
   D. 0%

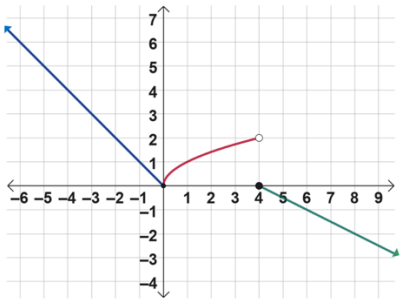
	4. What is the primary purpose of the Lifeline Rate Structure as explicitly mentioned in the passage? A. to encourage high-income households to consume more electricity B. to penalize consumers who exceed 100 kWh of power usage C. to cushion low-income households from rising energy costs D. to eliminate administrative fees for all electricity consumers 5. Based on the passage, why is a single standard algebraic equation ($y = mx + b$) insufficient for calculating a complete electric bill under the Lifeline system? A. because electric companies change their pricing structure every single day B. because electricity cannot be measured using real numbers C. because administrative fees are illegal under Philippine law D. because a single formula cannot accommodate changing mathematical rules across different consumption tiers 6. Why does the passage emphasize that kilowatt-hour consumption intervals must be *"non-overlapping"*? A. to ensure that a consumer's usage never falls into two different discount rules at the same time B. to make sure electric bills are as expensive as possible C. to allow power companies to charge consumers twice for the same electricity D. to prevent low-income households from receiving any discount 7. What can be inferred about a household that consumes 120 kWh of electricity in a billing cycle? A. They will receive a 100% discount on their entire bill B. They qualify for the 35% discount tier C. They fall outside the lifeline discount range and pay the standard rate D. Their electric bill cannot be calculated mathematically 8. Which mathematical concept directly mirrors the *"Lifeline Rate Structure"* described in the passage? A. a quadratic function with a single vertex B. a linear equation with a constant slope C. a piecewise function with multiple sub-functions defined over specific domain intervals D. an absolute value equation that turns all negative inputs into positive outputs 9. In the context of piecewise functions, how would the *"demarcation points"* (20 kWh, 50 kWh, 100 kWh) mentioned in the text be classified in a mathematical coordinate system? A. y-intercepts B. breakpoints or boundary values where sub-functions transition C. maximum turning points of a parabola D. asymptotes that the graph can never touch 10. Suppose you are asked to represent the Lifeline Rate Structure as a formal piecewise function $B(x)$, where x represents consumption in kWh. Which inequality correctly represents the domain condition for the 50% discount tier? A. $x \leq 0$ B. $x > 100$ C. $50 < x < 70$ $20 < x \leq 50$
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. When transitioning from evaluating a piecewise function algebraically to drawing its graph, how do domain conditions dictate both your calculation steps and your choice of boundary endpoints? 2. In the Representing Real-Life Situations and Reading Integration activities, you worked with tiered systems like tax brackets, water billing, and electricity lifeline rates. Why are piecewise functions the most effective mathematical tool for modeling these real-world scenarios instead of a single, continuous equation? 3. Looking back across all four activities, which step or transition was the most challenging for you (e.g., setting up inequalities from word problems, graphing non-linear sub-functions like square roots, or interpreting text-based domain rules)?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper calculator
III. Instructions:	*Choose the letter of the correct answer.* 1. Which of the following best describes a piecewise function? A. A function defined by different sub-functions, each applying to a specific part of the domain. B. A single continuous function defined by a single linear equation for all real numbers. C. A collection of distinct mathematical functions combined into a single system. D. An equation that gives two different output values (y) for every single input value (x). 2. Why is it impossible for a valid piecewise function to have domain conditions such as $x \leq 4$ and $x \geq 4$ applied to two different sub-functions? A. The graph will automatically become discontinuous at $x = 4$. B. The boundary $x = 4$ cannot be included in any algebraic interval. C. It violates the definition of a function by giving two potential outputs for $x = 4$. D. It prevents the function from having a slope at the breakpoint. 3. In the piecewise function $f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ 5, & \text{if } 0 \leq x \leq 5 \\ 2x-1, & \text{if } x > 5 \end{cases}$, which sub-function rule applies when evaluating $x = 0$? A. $x + 2$ B. $2x - 1$ C. $x + 2$ and 5 simultaneously D. 5 4. Given $g(x) = \begin{cases} \lvert x^2 - 12 \rvert, & \text{if } x < 2 \\ 2\sqrt{5x+6} - 1, & \text{if } x \geq 2 \end{cases}$, what is the value of $g(2)$? A. 7 B. 8 C. 9 D. 15 5. Evaluate $h(-3)$ using the function $h(x) = \begin{cases} 2x^2 + 3x - 1, & \text{if } x \leq -1 \\ 4, & \text{if } -1 < x < 3. \\ x - 2, & \text{if } x \geq 3 \end{cases}$ A. -10 B. 8 C. 4 D. 26 6. What is the value of $f(4)$ given $f(x) = \begin{cases} \lvert 2x - 10 \rvert + 3, & \text{if } x \leq 4 \\ 12 - x, & \text{if } 4 < x \leq 10 \end{cases}$? A. 12 B. 8 C. 5 D. 0 7. If $k(x) = \begin{cases} x^3, & \text{if } x < -2 \\ 2x + 10, & \text{if } -2 \leq x \leq 3 \\ 15 - x, & \text{if } x > 3 \end{cases}$, what is the sum of $k(-2) + k(4)$? A. 11 B. 5 C. 15 D. 17 8. When graphing a boundary point (breakpoint) defined by a strict inequality ($x < 5$ or $x > 5$), which symbol must be plotted on the graph? A. A closed solid circle (●) B. An open unshaded circle (○) C. A dashed vertical line D. An arrowhead (→) 9. Consider the graph of $f(x) = \begin{cases} x + 1, & \text{if } x \leq 1 \\ 4 - x, & \text{if } x > 1 \end{cases}$. What occurs at $x = 1$? A. There is an open circle at (1,2) and a closed circle at (1,3). B. Both line segments meet at (1,2) with a closed circle, making it continuous.

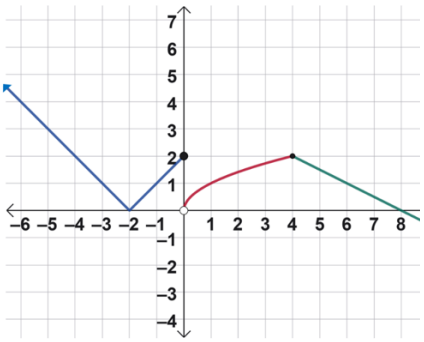
C. There is a jump gap making the graph discontinuous.
D. The graph ends abruptly and has no points defined for $x \geq 1$.

10. Which statement correctly describes the graph of $p(x) = \begin{cases} 2, & \text{if } x < 0 \\ -x + 2, & \text{if } 0 \leq x < 3 \\ 1, & \text{if } x \geq 3 \end{cases}$?
    A. It consists of three parallel lines with constant positive slope.
    B. It is continuous at $x = 0$ but has a jump gap (discontinuity) at $x = 3$.
    C. It has an open circle at (0,2) and a closed circle at (3,1).
    D. It is discontinuous at both $x = 0$ and $x = 3$.
11. Which of the following graphs correctly represents the piecewise function
    $f(x) = \begin{cases} |x + 2|, & \text{if } x < 0 \\ \sqrt{x}, & \text{if } 0 \leq x < 4 \\ -\frac{1}{2}x + 4, & \text{if } x \geq 4 \end{cases}$?

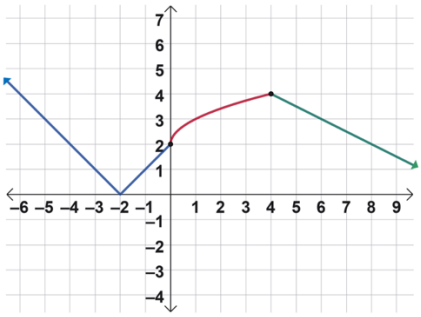
A.



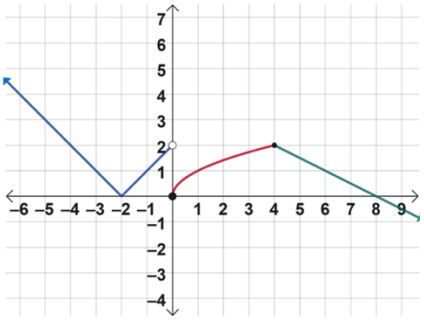
C.



B.



D.



12. A local parking lot charges ₱40 for the first 2 hours. Every additional hour (or fraction of an hour) beyond 2 hours costs an extra ₱15. Which function models the total cost $C(h)$ for parking h hours ($h > 2$)?
    A. $C(h) = 40 + 15h$
    B. $C(h) = 55(h - 2)$
    C. $C(h) = 40 + 15(h - 2)$
    D. $C(h) = 15 + 40(h - 2)$
13. A tricycle fare in a certain barangay charges a base rate of ₱13 for the first 4 kilometers. For distance d (in km) greater than 4 km, an additional ₱2 is charged per kilometer. Which is the correct formal piecewise function $F(d)$ for this fare structure?
    A. $f(d) = \begin{cases} 13, & \text{if } 0 < d \leq 4 \\ 13 + 2(d - 4), & \text{if } d > 4 \end{cases}$
    B. $f(d) = \begin{cases} 13, & \text{if } 0 \leq d < 4 \\ 2d, & \text{if } d \geq 4 \end{cases}$
    C. $f(d) = \begin{cases} 13d, & \text{if } 0 < d \leq 4 \\ 15d, & \text{if } d > 4 \end{cases}$
    D. $f(d) = \begin{cases} 13 + 2d, & \text{if } 0 < d \leq 4 \\ 15(d - 4), & \text{if } d > 4 \end{cases}$
14. An internet cafe offers a promo: ₱20 per hour for the first 3 hours ($0 < h \leq 3$). If a customer stays for more than 3 hours, a flat rate of ₱15 per hour applies to ALL hours spent. What will a student pay if they use a computer for 5 hours?
    A. ₱60
    B. ₱100

	C. ₱75 D. ₱90 15. A local bookstore sells customized notebooks where the pricing structure depends on the total number of notebooks (x) purchased in a single transaction. Under this policy, buying 1 to 5 notebooks costs ₱120 per notebook, purchasing 6 to 10 notebooks reduces the price to ₱110 per notebook, ordering 11 to 20 notebooks further discounts the rate to ₱100 per notebook, and buying more than 20 notebooks offers the lowest rate of ₱85 per notebook. Which of the following piecewise functions $C(x)$ correctly represents the total cost of purchasing x notebooks? A. $C(x) = \begin{cases} 120, & \text{if } 1 \le x \le 5 \\ 110, & \text{if} 6 \le x \le 10 \\ 100, & \text{if } 11 \le x \le 20 \\ 85, & \text{if } x > 20 \end{cases}$ B. $C(x) = \begin{cases} 120x, & \text{if } 1 \le x \le 5 \\ 110x, & \text{if} 6 \le x \le 10 \\ 100x, & \text{if } 11 \le x \le 20 \\ 85x, & \text{if } x > 20 \end{cases}$ C. $C(x) = \begin{cases} 120x, & \text{if } x < 5 \\ 110x, & \text{if} 5 \le x < 10 \\ 100x, & \text{if } 10 \le x < 20 \\ 85x, & \text{if } x \ge 20 \end{cases}$ D. $C(x) = \begin{cases} 120x, & \text{if } 1 \le x \le 5 \\ 110(x-5), & \text{if} 6 \le x \le 10 \\ 100(x-10), & \text{if } 11 \le x \le 20 \\ 85(x-20), & \text{if } x > 20 \end{cases}$
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. Looking back at the test items you found most challenging, was the difficulty in understanding the real-world scenario, applying the correct inequality/boundary symbol (○ vs. ●), or doing the algebra? How will you tackle that specific step next time? 2. When evaluating or graphing piecewise functions, why is it so important to check the domain condition (x-interval) before plugging in a number or drawing a line? What happens if you skip this step? 3. When turning a real-life situation (like a tiered rate or bill) into a piecewise function, how do you determine when a charge applies to the entire input versus only the amount exceeding a threshold? What steps do you take to avoid mixing these up?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials	pen and paper calculators
III. Instructions:	**For Remediation** *Determine which sub-function condition applies to the given value of x, then evaluate the function. Show your substitution step for each item.* $f(x) = \begin{cases} 2x+7, & \text{if } x < 3 \\ x^2 - 4, & \text{if } x \ge 3 \end{cases}$ $g(x) = \begin{cases} \lvert x+4 \rvert, & \text{if } x \le -2 \\ 3x+1, & \text{if } -2 < x < 4 \\ \sqrt{x+5}, & \text{if } x \ge 4 \end{cases}$ 1. $f(0)$ 2. $f(-5)$ 3. $f(3)$ 4. $f(5)$ 6. $g(-6)$ 7. $g(-2)$ 8. $g(0)$ 9. $g(3)$

	5. $f(2.5)$ 10. $g(11)$ **For Enhancement** Analyze each real-world scenario carefully and formulate the corresponding piecewise function. Pay close attention to cumulative fees, strict boundary thresholds, and conditional discounts. 1. A local water utility company charges a flat monthly fee of ₱180 for consumption up to 10 m^3. For consumption exceeding 10 m^3 up to 30 m^3, each additional cubic meter costs ₱25. For consumption above 30 m^3, a heavy-user rate applies: the consumer pays the total accumulative cost of the first 30 m^3 plus ₱40 for every cubic meter beyond 30 m^3. Write $W(x)$ representing the total cost for x m^3 of water, where $x \geq 0$. 2. An e-scooter app charges ₱30 to unlock the vehicle, which includes the first 15 minutes of riding for free. After 15 minutes, the ride costs ₱3 per minute for the next 45 minutes (up to 60 total minutes). If the rental exceeds 60 minutes, the customer pays a flat penalty fee of ₱100 plus ₱5 for *every* minute of the entire trip duration. Write $R(m)$ representing the total rental cost for m minutes, where $m > 0$. 3. A print shop charges ₱200 per t-shirt for orders of fewer than 20 shirts. For orders of 20 to 50 shirts, they offer a 10% discount on all shirts ordered. For orders exceeding 50 shirts, the first 50 shirts are charged at the 10% discounted rate, while any additional shirt beyond 50 gets a 25% discount off the original ₱200 base price. Write $P(n)$ representing the total price for printing n shirts.
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. Comparing how you felt during today's assessment to how you felt while working on these extra practice problems, which specific type of problem became easier for you to solve, and why? 2. What was one tricky detail or common trap you caught yourself watching out for while setting up your equations or evaluating function inputs? 3. If you had to teach a classmate how to set up the boundary intervals (like $0 < x \leq 10$ vs. $x > 10$) without overlapping them, what simple rule or tip would you share with them?

Unit 7. Data and Measures of Central Tendency			
Name:		**Grade & Section:**	

Lesson 7.1. Data and Its Level of Measurement	
Objectives:	At the end of the lesson, you should be able to: 1. identify types of data; 2. classify data according to its type; 3. identify levels of measurement; and 4. classify data according to its level of measurement.
Key Ideas and Examples:	**Exploring Types of Data** **Types of Data and Levels of Measurement** In statistics, *data* refers to any collection of facts, figures, or information gathered for the purpose of analysis. These data are essential because they help us understand patterns, make decisions, and draw meaningful conclusions. Before analyzing data, it is important to first identify its *type* and its *level of measurement*, since these determine the appropriate statistical tools and methods to use. **I. Types of Data** Data are generally classified into two main types: *qualitative data* and *quantitative data*. **A. Qualitative Data (Categorical Data)** *Qualitative data* describe characteristics, qualities, or categories. These data are not expressed in numbers and cannot be used for mathematical computations. Instead, they are used to group or label information. ***Definition:*** Qualitative data are non-numerical data that represent attributes, labels, or categories. ***Key Features:*** ● Descriptive in nature ● Expressed using words, symbols, or labels ● Used for classification ● Cannot be meaningfully added, subtracted, multiplied, or divided ***Examples:*** ● Hair color: black, brown, blonde ● Gender: male, female ● Type of school: public, private ● Marital status: single, married, widowed ● Favorite food: rice, pizza, noodles **B. Quantitative Data (Numerical Data)** *Quantitative data* involve numbers that represent counts or measurements. These data can be analyzed using mathematical operations such as addition, subtraction, multiplication, and division. ***Definition:*** Quantitative data are numerical values that represent measurable quantities. ***Key Features:*** ● Expressed in numbers ● Can be measured or counted ● Allows mathematical computation ● Used for statistical analysis ***Examples:*** ● Age: 16 years old ● Number of learners in a class: 45 ● Weight: 52.5 kg ● Income: ₱25,000 per month ● Number of books read: 10

II. Levels of Measurement

After identifying whether data are qualitative or quantitative, the next step is to determine their *level of measurement*. This is important because it tells us how data can be organized, compared, and analyzed statistically.

There are *four levels of measurement*, arranged from the simplest to the most advanced: *nominal, ordinal, interval, and ratio*.

A. Nominal Level (Categorical Level)

Definition:

The nominal level classifies data into distinct categories without any order or ranking.

Key Features:

- Used for labeling or naming
- No meaningful order
- No mathematical operations possible
- Purely categorical

Examples:

- Blood type: A, B, AB, O
- Type of fruit: apple, banana, mango
- Region: NCR, CAR, Region IV-A
- Eye color: brown, black, blue

B. Ordinal Level (Ranked Data)

Definition:

The ordinal level involves data that can be arranged in a meaningful order or ranking, but the differences between ranks are not equal or measurable.

Key Features:

- Shows order or ranking
- Differences between ranks are not exact
- No precise measurement of distance between values

Examples:

- Survey rating: satisfied, neutral, dissatisfied
- Class ranking: 1st place, 2nd place, 3rd place
- Socioeconomic status: low, middle, high
- Academic honors: with honors, with high honors, with highest honors

C. Interval Level

Definition:

The interval level includes numerical data with meaningful differences between values. However, it has *no true zero point*, meaning zero does not represent the absence of the quantity.

Key Features:

- Numerical data
- Equal intervals between values
- No true zero
- Addition and subtraction are meaningful

Examples:

- Temperature in Celsius: 20°C, 30°C
- Calendar years: 2000, 2010, 2020
- IQ scores: 90, 100, 110

D. Ratio Level

Definition:

The ratio level is the highest level of measurement. It has all the characteristics of the interval level, but with a *true zero*, which means zero represents the absence of the quantity.

Key Features:

- Numerical data
- Equal intervals
- Has a true zero point
- Allows meaningful ratios (e.g., twice as much)

	Examples: ● Height: 160 cm ● Weight: 60 kg ● Age: 18 years old ● Income: ₱30,000 per month ● Number of students: 0, 25, 50 Understanding the *type of data* (qualitative or quantitative) and the *level of measurement* (nominal, ordinal, interval, or ratio) is essential in statistics. These classifications guide researchers and students in selecting appropriate methods for organizing, analyzing, and interpreting data accurately.
Other Resources: *(Digital Modules and Textbook, Video Lessons)*	Belecina, R. R., Baccay, E. S., and Mateo, E.B. (2016). *Statistics and Probability, 1st Edition*. Quezon City: Rex Book Store, Inc. pp. 21-45 Bluman, A. (2015). *Elementary Statistics: A Step-by-Step Approach*. London: McGraw-Hill Education - Europe. pp. 258-274 Brase, C. H. and Brase, C. P. (2004, 2019). *Understanding Basic Statistics 3rd and 8th editions*. Boston, USA: Cengage Learning. pp. 246-258 Dela Cruz, E. O., et al. (2016). *Soaring 21st Century Mathematics: Statistics and Probability (K to 12).* Quezon City: Phoenix Publishing House, Inc. pp. 16-25

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen and paper
III. Instructions:	*Carefully read each scenario in the table, determine whether the data is* qualitative *or* quantitative*, identify its appropriate* level of measurement *(nominal, ordinal, interval, or ratio), and provide a brief and clear justification for your answer in the last column. Be prepared to discuss and justify your answers in class.*

Scenario	Type of Data	Level of Measurement	Reason
1. Ranking of top 5 learners based on academic performance			
2. Daily temperature recorded in Celsius for a week from PAGASA			
3. Number of hours spent on social media per day			
4. Preferred learning modality (modular, online, F2F, blended) of learners			
5. Amount of rice consumed by a household per week (in kg)			
6. The modes of transportation learners use to go to school (e.g., tricycle, jeep, walking, bicycle)			
7. Monthly electricity consumption of your household in kilowatt-hours			
8. The type of music learners listen to while studying			
9. Number of correct items the student gets in a quiz			
10. Learners' satisfaction with school library using a 5-point scale (1 = Very Dissatisfied, 5 = Very Satisfied)			

V. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. How did you determine whether each scenario represents qualitative or quantitative data? 2. What clues helped you identify the correct level of measurement for each item? 3. Which level of measurement was easiest to identify? Why?

	4. Which items were most challenging to classify? What made them difficult? 5. How can understanding data classification help in interpreting real-life information correctly?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper
III. Instructions:	*Complete the table below by identifying whether each data set is* qualitative *or* quantitative *and its* level of measurement *(nominal, ordinal, interval, or ratio).*

Scenario	Type of Data	Level of Measurement
1. Collecting the shoe sizes of volleyball players on the school team.		
2. Listing the favorite local fruits (e.g., mango, banana, lanzones) of market vendors.		
3. Recording monthly electricity bills (in pesos) from 10 households.		
4. Ranking the cleanliness of streets in the barangay from most to least clean.		
5. Measuring the time (in minutes) spent by learners traveling to school each day.		
6. Listing the types of pets owned by learners (e.g., dog, cat, bird, fish).		
7. Recording the temperature (°C) inside classrooms throughout the day.		
8. Counting how many plastic bottles are collected for recycling each week.		
9. Categorizing different brands of detergents used in households.		
10. Recording the ratings given by residents on barangay services from 1 (poor) to 5 (excellent).		

IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. How did you determine whether each scenario represents qualitative or quantitative data? 2. What key indicators helped you identify the correct level of measurement? 3. Which items were easiest to classify? Why? 4. Which items were most challenging? What made them difficult to identify? 5. How does understanding data classification help in interpreting real-life situations more accurately?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials	pen and paper
III. Instructions:	**For Remediation** *Read each scenario carefully. Identify whether the data is* qualitative *or* quantitative*, determine its correct* level of measurement *(nominal, ordinal, interval, or ratio), and record your answers in the table provided.*

Scenario	Type of Data	Level of Measurement
1. The color of school uniform		
2. The number of siblings each student has		
3. Ratings given to a barangay clean-up program (Very Poor to Excellent)		
4. Average height (in cm) of Grade 10 learners		
5. Student ID numbers assigned to each student		

For Enhancement

Read the given research title and statement of the problem carefully. Identify the possible data to be collected for each research question, classify each as qualitative *or* quantitative*, determine its corresponding* level of measurement *(nominal, ordinal, interval, or ratio), and present your answers in the table provided.*

Title: A Study on the Study Habits and Academic Performance of Senior High School Learners

Statement of the Problem: This study aims to determine the relationship between learners' study habits and their academic performance.

Research Question / Data to be Collected	Type of Data	Level of Measurement
1. Preferred study times (morning, afternoon, evening)		
2. Number of hours spent studying each day		
3. Final grades in Mathematics for the last grading period		
4. Rank of learners based on overall academic performance		

IV. Reflection:

To help you reflect on your learning experience, answer the following feedback and reflection questions

1. How did you identify whether each scenario represents qualitative or quantitative data?
2. What helped you determine the correct level of measurement for each item?
3. Which items in the remediation task were easier or more difficult to classify? Why?
4. How does the enrichment task connect data classification to real research studies?
5. Why is it important to correctly classify data before analyzing research results?

Lesson 7.2. Understanding Measures of Central Tendency

Objectives:

By the end of the lesson, you will be able to:

1. identify the appropriate measure of central tendency of a given data set for different purposes;
2. use appropriate technologies (e.g. calculator, Excel, Jamovi, Geogebra, etc.) to calculate measures of central tendency; and
3. interpret the computed measures of central tendency.

Key Ideas and Examples:

Understanding Each Measure of Central Tendency

Measures of central tendency are statistical tools used to describe the center or typical value of a set of data. They provide a single value that represents the entire distribution, allowing us to make generalizations, comparisons, and informed decisions based on data.

The three most common measures are:

- Mean – also called the average
- Median – the middle value when data is ordered
- Mode – the most frequently occurring value

Each has specific uses depending on the nature and distribution of the data.

Definition and Application of Each Measure

A. Mean (Average)

Definition:

- The mean is the sum of all values divided by the number of values.

Application:

- Used when data is uniform and free from extreme values (outliers). It is commonly used in computing averages like grades, temperatures, and

performance scores.

Strengths:

- Uses all data values, providing a balanced summary.
- Widely understood and accepted.

Weaknesses:

- Sensitive to outliers; extreme values can distort the result.
- May not represent the typical value in a skewed distribution.

A **skewed distribution** is a lopsided data set. Instead of forming a perfectly balanced, symmetrical bell curve where most data sits right in the middle, the data is pulled or stretched toward one extreme side, creating a long "tail."

The Two Types

1. Right-Skewed (Positive)

- **The Shape:** A big clump on the left; the long tail stretches to the **right**.
- **Example:** Housing prices. Most houses are average-priced, but a few multi-million dollar mansions pull the **Mean** way up.
- **Order: Mean > Median > Mode**

2. Left-Skewed (Negative)

- **The Shape:** A big clump on the right; the long tail stretches to the **left**.
- **Example:** Age at retirement. Most people retire in their 60s or older, but a few people retiring very early pull the **Mean** down.
- **Order: Mean < Median < Mode**

Example:

The quiz scores of 8 learners in a 10-item quiz are:

9, 8, 7, 10, 9, 8, 6, 10

Mean: **8.375**

Interpretation:

The mean score is 8.375, which suggests that, on average, learners performed well on the quiz. Since the scores are fairly close to each other and there are no extreme outliers, the mean effectively represents the central performance of the group. Most of the individual scores (6, 7, 8, 9, and 10) revolve around or are near the mean, indicating a consistent level of achievement among the learners.

B. Median

Definition:

- The median is the middle value of an ordered data set. If there's an even number of values, the median is the average of the two middle values.

Application:

- Best used when data is skewed or has outliers, such as income distributions or test scores with very low or high extremes.

Strengths:

- Unaffected by outliers, making it a reliable measure in skewed data.
- Useful when the exact value at the center is important.

Weaknesses:

- Does not use all values, so some information may be ignored.
- Not suitable for nominal data.

Example:

Consider the scores of 9 learners:

45, 48, 50, 50, 50, 52, 55, 97, 100

Median: **50**

Interpretation:

Although one student scored 97 and the other one scored 100, the median score is 50, meaning that half of the learners scored below 50 and half scored above 50 and half scored above 50. The median gives a better representation of the overall performance because it is not affected by the two unusually high scores.

	C. Mode *Definition:* • The mode is the value that appears most frequently in a data set. *Application:* • Best used with categorical or nominal data (e.g., favorite color, shoe size) or when identifying the most common item is important. *Strengths:* • The only measure that can be used with qualitative data. • Easy to identify and interpret. *Weaknesses:* • A data set may have no mode, one mode, or multiple modes, which can complicate interpretation. • Ignores most data values; may not reflect the center well. *Example:* The shirt sizes sold in a store in one week: M, L, M, XL, M, S, L, M Step 1: Tally the frequencies: S = 1 M = 4 L = 2 XL = 1 Mode = M *Interpretation*: The mode is M, indicating that medium-sized shirts were the most commonly sold. This is useful for inventory planning, as it reveals customer preference.
Other Resources: *(Digital Modules and Textbook, Video Lessons)*	Bluman, A. (2015). *Elementary Statistics: A Step-by-Step Approach.* London: McGraw-Hill Education - Europe. pp. 258-274 Brase, C. H. and Brase, C. P. (2004, 2019). *Understanding Basic Statistics 3rd and 8th editions*. Boston, USA: Cengage Learning. pp. 246-258 Dela Cruz, E. O., et al. (2016). *Soaring 21st Century Mathematics: Statistics and Probability (K to 12).* Quezon City: Phoenix Publishing House, Inc. pp. 16-25 Department of Education (2013). *Mathematics 8: Module 10 – Measures of Central Tendency and Measures of Variability*. DepEd. https://lrmds.deped.gov.ph/detail/4772

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen and paper calculator or laptop
III. Instructions:	*Read each scenario carefully. Determine the most appropriate measure of central tendency (M), then compute its value either manually or using statistical software (e.g., Excel formulas or the Jamovi Exploration panel). Complete the table by providing the Appropriate Measure, Reason, Computed Value, and Interpretation. Be prepared to share and justify your analysis during the class review or submit your completed data charts to your teacher's grading ledger.* 1. A science teacher recorded the scores of 30 learners in a 100-point test. The scores were: *Scores*: 86, 90, 88, 92, 89, 87, 85, 91, 90, 89, 88, 87, 86, 90, 88, 92, 89, 87, 88, 91, 85, 89, 90, 88, 87, 89, 91, 86, 88, 90 a. Appropriate Measures of Central Tendency: b. Reason: c. Computed value: d. Interpretation:

	2. A public school principal is analyzing the monthly family income (in PHP) of 31 learners to understand economic disparities: Monthly Incomes (in PHP): 6,500; 7,000; 7,500; 8,000; 8,500; 9,000; 9,500; 10,000; 10,500; 11,000; 11,500; 12,000; 12,500; 13,000; 13,500; 14,000; 14,500; 15,000; 15,500; 16,000; 16,500; 17,000; 17,500; 18,000; 18,500; 19,000; 19,500; 20,000; 21,000; 23,000; 120,000 a. Appropriate Measures of Central Tendency: b. Reason: c. Computed value: d. Interpretation: 3. A PE teacher records the shoe sizes of 40 learners: Shoe Sizes: 7, 8, 8, 9, 8, 7, 8, 9, 10, 8, 7, 9, 8, 8, 9, 7, 8, 9, 10, 8, 8, 7, 8, 9, 8, 9, 10, 8, 7, 8, 8, 9, 7, 8, 9, 10, 8, 7, 8, 9 a. Appropriate Measures of Central Tendency: b. Reason: c. Computed value: d. Interpretation:
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. When computing the family income in Problem 2, how did your software's automated calculation help you spot the danger of using the mean instead of the median without having to do long addition by hand? 2. Look at Problem 2 and Problem 3. If a government group used the *mean* income to distribute school financial aid, or if a shoe factory used the *mean* shoe size to manufacture products, what real-world problems would happen?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper calculator or laptop
III. Instructions:	*Read the following problems and answer what is asked. Use manual computation, Excel or Jamovi in the computation process.* 1. A call center supervisor recorded the number of minutes it took 30 customer service agents to resolve tickets in one day. Response Times (in minutes): 12, 15, 13, 14, 16, 13, 12, 15, 14, 13, 16, 17, 13, 15, 14, 12, 15, 13, 14, 15, 16, 13, 14, 15, 14, 13, 16, 15, 14, 15 e. Appropriate Measures of Central Tendency: f. Reason: g. Computed Value: h. Interpretation: 2. A teacher surveyed 31 learners about their weekly allowance (in Php) to understand spending habits. Weekly Allowances (in Php) 50, 60, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195, 200, 210, 220, 1000 a. Appropriate Measures of Central Tendency: b. Reason: c. Computed Value: d. Interpretation: 3. An adviser recorded the number of times 40 learners raised their hands to answer questions during a week. Participation Counts: 2, 3, 3, 4, 3, 2, 3, 4, 5, 3, 2, 4, 3, 3, 4, 2, 3, 4, 5, 3, 3, 2, 3, 4, 3, 4, 5, 3, 2, 3, 3, 4, 2, 3, 4, 5, 3, 2, 3, 4 a. Appropriate Measures of Central Tendency: b. Reason: c. Computed Value: d. Interpretation:

IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. In Problem 2 (Weekly Allowances), how did the extreme value of Php 1,000 affect the dataset, and what would happen if we used the mean instead of the median? 2. When working with discrete, heavily repeated numerical data like the Participation Counts in Problem 3, why is the mode a useful measure compared to the mean? 3. If you were to enter these three datasets into Excel or Jamovi, what visual indicators (like graphs) could you use alongside central tendency to quickly spot outliers or data distribution? 4. Based on the results of the three problems, guide us on how to create a general rule for choosing the best measure of central tendency.

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials	pen and paper calculators
III. Instructions:	**For Remediation** *Analyze each scenario and determine the most appropriate measure of central tendency to obtain meaningful information. Compute the required value either manually or by using a statistical application (e.g., Excel or Jamovi).* 1. A restaurant manager recorded the total daily sales (in PHP) over 30 days to monitor business performance. Daily Sales (in PHP): 5,300; 5,500; 5,600; 5,800; 5,900; 6,000; 6,100; 6,300; 6,400; 6,500; 6,600; 6,700; 6,800; 7,000; 7,100; 7,300; 7,500; 7,600; 7,700; 7,800; 7,900; 8,000; 8,100; 8,300; 8,500; 8,600; 8,700; 8,800; 9,000; 25,000 Appropriate Measures of Central Tendency: Reason: Computed value: Interpretation: 2. A PE teacher tallied the number of sit-ups completed by 40 learners in one minute. Sit-Up Counts: 30, 32, 30, 28, 30, 29, 30, 31, 30, 29, 28, 30, 30, 32, 30, 30, 29, 28, 30, 30, 31, 30, 29, 30, 32, 30, 30, 28, 29, 30, 30, 30, 30, 29, 30, 31, 30, 28, 30, 30 Appropriate Measures of Central Tendency: Reason: Computed value: Interpretation: 3. A guidance counselor asked 31 learners how many hours of sleep they usually get each night. Sleep Hours: 4, 5, 5, 6, 6, 6, 6.5, 7, 7, 7.5, 8, 8, 8, 8, 8.5, 9, 9, 9, 9, 9.5, 10, 10, 10, 10.5, 11, 11.5, 12, 12, 12.5, 13, 14 Appropriate Measures of Central Tendency: Reason: Computed value: Interpretation: **For Enhancement** *Analyze each scenario and determine the most appropriate measure of central tendency to obtain meaningful information. Compute the required value either manually or by using a statistical application (e.g., Excel or Jamovi).* 1. A gardener buys 10 packets of seeds from two different companies. Each pack contains 20 seeds and he records the number of plants which grow from each pack.

Company A	20	5	20	20	20	6	20	20	20	8
Company B	17	18	15	16	18	18	17	15	17	18

	a. Find the mean, median, and mode for each company's seeds. b. Which company has the higher mean? c. Which company do you think is better with regard to seeds production? 2. Given the data set: 2, 4, 4, 6, 8, what will happen to the mean, median, and mode if 3 is added to each value? How do they compare to the original? 3. Given the data set: 1, 3, 3, 5, 7, what will happen to the mean, median, and mode when 2 is multiplied to each value?
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. How does the presence of an outlier affect the choice between using the mean and the median? 2. If a dataset is perfectly symmetrical, what can you predict about its mean, median, and mode? 3. Why is it beneficial to understand data transformation properties (like adding or multiplying a constant to a dataset)?

Lesson 7.3. Measures of Variability	
Objectives:	By the end of the lesson, you will be able to: 1. define and differentiate the different measures of variability; 2. compute the different measures of variability that include solving real-life; applications manually or by using open-source statistical software (e.g. MS Excel, Jamovi); and *3.* analyze and interpret the computed measures of variability.
Key Ideas and Examples:	**Understanding the Measures of Variability** a. *Range (R)* Range tells the spread of the data that involves the lowest and highest value. It is the simplest measure of variability by calculating the difference between the highest and lowest values in a dataset. It is particularly helpful in situations where understanding extreme values is important. Formula: R = Highest Value (HV) – Lowest Value (LV) *Example 1*: In a certain test in General Mathematics, the 40 Grade 11 learners scored from 14 to 39. Hence, the **range is 25** (i.e. R = 39 – 14 = 25). *Example 2*: Given the following data on the number of minutes to solve a math problem: {23, 10, 18, 19, 43, 16}. The **range is 33** (i.e. R = 43 – 10 = 33). **Note:** While range is easy to compute, it does not account for how values are distributed across a dataset. b. *Interquartile Range (IQR)* The interquartile range represents the spread of the middle 50% of the data. It is obtained by computing the difference between the third quartile (Q3) and first quartile (Q1) or between the 75^{th} percentile and 25^{th} percentile. It is useful when we find to measure the variability while minimizing the impact of the extreme values. Formula: IQR = Third quartile (Q3) – First quartile (Q1) = 75^{th} percentile – 25^{th} percentile *Example 3*: Given the following sample test scores of 10 Grade 11 learners in General Mathematics: {35, 32, 37, 45, 15, 23, 26, 26, 39, 40}. *Solution*: Step 1: Arrange the data in ascending order.: {15, 23, 26, 26, 32, 35, 37, 39, 40, 45}. Step 2: Determine Q1and Q3. (Q1 is 26 and Q3 is 38.5) Step 3: IQR = Q3−Q1 = 39-26= 12.5 Thus, the Interquartile Range (IQR) is 13. This means that the middle 50% of learners' test scores in General Mathematics are spread across 12.5 points. While test scores do vary, they are not extremely scattered.

Note: Since IQR is not affected by the extreme values, it is a resistant measure of variability. It tells us how spread out the central portion of the data is, without being affected by the outliers.

c. Variance and Standard Deviation

Variance represents the mean of squared differences from the average, while **Standard Deviation (SD)** is the square root of the variance. Both measures how spread out the data are from the mean.

There are two types of Variance/Standard deviation: *sample variance* (s^2) and *population variance* (σ^2); *sample Standard Deviation* (s) and *population standard deviation* (σ); A higher variance or standard deviation means the data are more spread out, while a lower variance means they are closer to the average.

Sample Variance and Sample Standard Deviation

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

where n is the sample size and $\underline{x}$ is the sample mean.

Population Variance and Population Standard Deviation

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x-\mu)^2}{N}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x-\mu)^2}{N}}$$

where N is the population size and μ is the population mean.

Note: Use **sample variance and sample standard deviation** when the data represent only a sample of a larger population. Use **population variance and population standard deviation** when the data include every member of the population being studied.

Example 4: Given the following sample test scores of 10 Grade 11 learners in General Mathematics in Example 3: {35, 32, 37, 45, 15, 23, 26, 26, 39, 40},

X	$X-\mu$	$(X-\mu)^2$
35	3.2	10.24
32	0.2	0.04
37	5.2	27.04
45	13.2	174.24
15	-16.8	282.24
23	-8.8	77.44
26	-5.8	33.64
26	-5.8	33.64
39	7.2	51.84
40	8.2	67.24
$\mu = \frac{\sum_i^N X}{N} = \frac{318}{10}$=31.8	$\sum_1^N (X-\mu)^2$	757.6
$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x-\mu)^2}{N}}$		$= \sqrt{\frac{757.6}{9}} = 9.17$

	The standard deviation of the given population is 9.17. This indicates that the performance of learners in General Mathematics vary moderately from the average score. Test scores are somewhat scattered, meaning learners didn't perform identically. Some learners scored higher or lower than others.
Other Resources: *(Digital Modules and Textbook, Video Lessons)*	Bluman, A. (2015). *Elementary Statistics: A Step-by-Step Approach.* London: McGraw-Hill Education - Europe. pp. 258-274 Brase, C. H. and Brase, C. P. (2004, 2019). *Understanding Basic Statistics 3rd and 8th editions*. Boston, USA: Cengage Learning. pp. 246-258 Dela Cruz, E. O., et al. (2016). *Soaring 21st Century Mathematics: Statistics and Probability (K to 12).* Quezon City: Phoenix Publishing House, Inc. pp. 16-25 Department of Education (2013). *Mathematics 8: Module 10 – Measures of Central Tendency and Measures of Variability*. DepEd. https://lrmds.deped.gov.ph/detail/4772

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen and paper calculator or laptop
III. Instructions:	*Read the scenario below and perform the indicted tasks..* *Scenario*: At ABC Senior High School, a General Mathematics teacher administered a 30-item quiz to two Grade 11 sections after discussing the lesson on measures of variability. After checking the quiz papers, the teacher found that both sections obtained nearly the same class average. However, the teacher realized that the average score alone could not determine whether the learners in each section performed with the same level of consistency. One section might have scores that were closely grouped around the average, while the other might have scores that were more widely spread. To analyze further, the teacher randomly selected the quiz scores of 10 learners from each section to serve as samples for statistical analysis. The teacher planned to compare the variability of the two sections by computing the Range, Interquartile Range (IQR), Variance, and Standard Deviation. These measures would help determine which section showed more consistent performance and which section had greater differences among learners' scores. The quiz scores of the randomly selected learners are shown below.

Section A (Sample of 10 Learners)	Section B (Sample of 10 Learners)
25	18
32	20
20	21
15	19
18	23
10	24
25	20
28	22
24	27
21	30

Using manual computation, Microsoft Excel, or Jamovi, help the General Mathematics teacher analyze the quiz results by computing the Range, Interquartile Range (IQR), Sample Variance, and Sample Standard Deviation for each section. Compare the measures of variability of the two sections to determine which section demonstrates more consistent learner performance, and interpret what the results reveal about the spread of the scores. After completing the computations, answer the guide questions that follow to analyze, interpret, and apply your findings.

IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. What assessment was given, why were 10 learners from each Grade 11 section selected, and why is comparing class averages alone not enough to determine which section performed more consistently? 2. Using the Range, Interquartile Range (IQR), Variance, and Standard Deviation, which Grade 11 section showed a wider spread of scores and which performed more consistently? What does a larger standard deviation imply about learners' quiz scores? 3. Why is it important to verify manual computations using Microsoft Excel and Jamovi, which method did you find most efficient, and what would it mean if another pair of Grade 11 sections had the same average score but a much larger standard deviation?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper calculator or laptop
III. Instructions:	*Read each scenario carefully. Determine and compute the appropriate measures of dispersion—**range, interquartile range (IQR), variance, and standard deviation**—using either manual calculations or statistical software (e.g., Excel or Jamovi). Record the computed values, provide a brief interpretation of each measure, and be prepared to explain and justify your results during the class discussion or submit your completed analysis to your teacher's grading ledger.* 1. Scenario: A group of Grade 11 learners receives a weekly allowance (in pesos) from their parents. They collect data from 30 Grade 11 learners over one week: 700 850 750 1000 1150 450 500 1100 1200 1250 1300 650 950 1000 1500 2000 450 850 650 550 1250 525 800 750 650 1250 450 500 650 1350 Task: a. Calculate the range, interquartile range, variance, and standard deviation b. What can you say about the variance and standard deviation? c. Which of the measures of variability is considered reliable for the scenario? Explain your answer. d. What happens to the measures of variability if we add or multiply a constant to each observation in a data set? 2. Scenario: Your school notices that learners struggle with concentration during peak heat index hours. Teachers track heat index variability to determine whether afternoon classes should be adjusted. Over the 30-day observation, the following data were recorded. 23 33 39 36 34 35 25 32 34 37 35 36 26 30 35 34 32 35 27 31 32 29 31 34 28 37 33 30 33 37 Task: a. Find the range of the dataset. b. Compute the interquartile range. c. Compute the variance and standard deviation. d. Why is it important to measure the variability of the heat index rather than just average values? e. What happens to the measures of variability if we add or multiply a constant to each observation in a data set?

	3. Scenario: A teacher wants to assess the consistency of learner performance in a General Mathematics exam. He/she collects scores from a class of 40 learners and groups them into intervals. The data are given below. 41 43 22 21 18 23 35 34 32 33 24 14 15 23 32 37 37 36 34 35 33 32 21 18 19 20 21 22 23 21 21 23 24 25 27 18 19 29 19 30 Task: a. Find the range, interquartile range, variance, and standard deviation. b. Is Range the most appropriate measures of variability for the given data? Why or why not? How about the interquartile range? How about the variance? How about the standard deviation? Explain your answer. c. What happens to the measures of variability if we add or multiply a constant to each observation in a data set? 4. Scenario: A teacher records the daily screen time (in hours) for 30 learners over a week. The learners' screen time varies based on study habits, gaming, and social media use. The data are given below. 5 6 3 9 5 7 4 10 8 6 6.5 7.5 8.5 9 7.5 6.5 5.5 4.5 9 8 7.5 7.5 8 10 9 3 4 4.5 5.5 10.5
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. What does a large or small value of the variance and standard deviation tell you about the data? 2. Among the range, interquartile range, variance, and standard deviation, which measure of variability is the most appropriate for your dataset? Why? 3. How can measures of variability help us make better decisions in real-life situations?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials:	pen and paper calculator or laptop
III. Instructions:	**For Remediation** *Read each problem carefully and perform the indicated tasks.* 1. A water utility company wants to analyze household water consumption variability to improve resource allocation and promote conservation. Below is the collected data of 20 households on their water consumption (in liters). 750 760 560 430 500 480 670 340 650 900 880 530 780 850 780 610. 670 950 670 820 a. Calculate the following measures of variability. • range • interquartile range • variance • standard deviation b. Interpret the results.

2. Two groups of learners were asked about their mobile data consumption (i.e. Group 1 learners – mobile data usage for academic purposes and Group 2 learners – mobile data usage for social media purposes). The collected data on monthly mobile data consumption (in gigabytes) among 20 learners (10 for each group) are as follows.

Group 1	
9	10.5
10	9.5
8.5	7.5
9.5	13
12	11.5

Group 2	
8	7.5
12	9
13	10
16	10.5
6.5	11

a. For each group, calculate variability.
   - range
   - interquartile range
   - variance
   - standard deviation

b. Interpret the results.

c. Which group shows more consistent mobile data usage? Which group has more extreme fluctuations?

d. Which group shows more consistent mobile data usage? Which group has more extreme fluctuations?

For Enhancement

Read the problem carefully and perform the indicated tasks

Gen and Mat own a small milktea shop and want to analyze the daily sales of three best-selling flavors: Black Milktea and Taro Milktea. They recorded the number of cups sold over 10 days:

Black Milktea	Taro Milktea	Thai Milktea
32	31	32
32	32	33
34	34	34
35	35	36
31	36	30
31	23	28
33	37	21
35	34	39
28	31	37
32	28	22

1. Find the range, interquartile range, variance, and standard deviation for each milktea flavor.
2. Interpret the computed measures of variability.
3. Which flavor has the largest variability in sales? How about the smallest variability in sales? Explain your answer.
4. How would adding data from another 10 days change the overall variability?
5. What happens to the measures of variability If we add 5 and multiply 2 to each observation in the given data set?
6. How can understanding variability help them improve customer satisfaction?

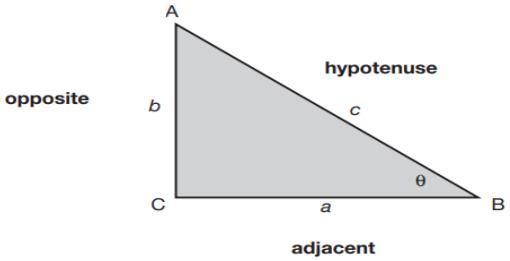
IV. Reflection:

To help you reflect on your learning experience, answer the following feedback and reflection questions

1. After computing the measures of variability, what do the results tell you about the consistency of the data?
2. Why is it important to compare the variance and standard deviation when analyzing two or more datasets?
3. How can measures of variability help people make better decisions in real-life situations such as water consumption, mobile data usage, or business sales?

Unit 8. Right and Oblique Triangles			
Name:		**Grade & Section:**	

Lesson 8.1. Trigonometric Ratios in a Right-Angled Triangle Using a Real-Life Object or Figure	
Objectives:	At the end of the activity, you should be able to: 1. find the unknown sides or angles in a right triangle; and 2. use Trigonometric ratios to solve problems involving a real-life object or figure.
Key Ideas and Examples	**Concept of Six Trigonometric Ratios** *Instructions*: To summarize and establish what you have learned from the previous activities, listen and participate to the discussions. In a right triangle, we can define six trigonometric ratios. Consider the right triangle ABC below. In this triangle we let θ represent ∠B. Then the leg denoted by a is the side adjacent to θ, and the leg denoted by b is the side opposite to θ.



We will use the convention that angles are symbolized by capital letters, while the side opposite each angle will carry the same letter symbol, in lowercase.

$$sin\ of\ \theta = sin\ \theta = \frac{opposite}{hypotenuse}$$
$$cos\ of\ \theta = \theta = \frac{adjacent}{hypotenuse}$$
$$tan\ of\ \theta = tan\ \theta = \frac{opposite}{adjacent}$$
$$\csc of\ \theta = csc\ \theta = \frac{hypotenuse}{opposite}$$
$$sec\ of\ \theta = sec\ \theta = \frac{hypotenuse}{adjacent}$$
$$\cot of\ \theta = cot\ \theta = \frac{adjacent}{opposite}$$

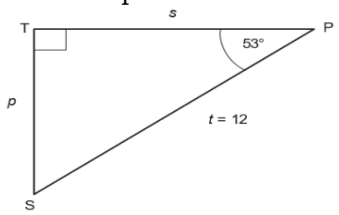
SOH-CAH-TOA is a mnemonic used for remembering the equations.

Notice that the three new ratios at the right are reciprocals of the ratios on the left. Applying algebra shows the connection between these functions.

$$csc\ \theta = \frac{hypotenuse}{opposite} = \frac{1}{\frac{opposite}{hypotenuse}} = \frac{1}{sin\ \theta}$$
$$sec\ \theta = \frac{hypotenuse}{adjacent} = \frac{1}{\frac{adjacent}{hypotenuse}} = \frac{1}{cos\ \theta}$$
$$cot\ \theta = \frac{adjacent}{opposite} = \frac{1}{\frac{opposite}{opposite}} = \frac{1}{tan\ \theta}$$

Showing a formula for the Missing Parts of a Right Triangle

Example 1: Determine the equation or formula to find a missing part of the triangle.



a. Solve s in the figure 1 above.
Solution: ∠P is an acute angle, t is the hypotenuse, s is the side adjacent to ∠P. Use CAH, that is

$$cos\ \theta = \frac{adajacent}{hypotenuse}$$
$$cos\ P = \frac{s}{t}$$
$$cos\ 53° = \frac{s}{t} = \frac{s}{12}$$
$$s = 12\ cos\ 53°$$

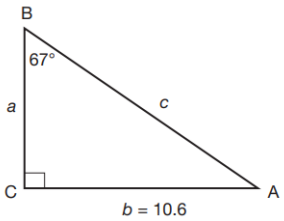
By using scientific calculator, key in 12 cos 53°
s = 7.2218

b. Solve for p in the figure above.
Solution: ∠P is an acute angle, t is the hypotenuse, s is the side adjacent to ∠P. Use SOH, that is

$$sin\ \theta = \frac{opposite}{hypotenuse}$$
$$P = \frac{p}{t}$$
$$sin\ 53° = \frac{p}{t} = \frac{p}{12}$$
$$s = 12\ sin\ 53°$$

By using scientific calculator, key in 12 sin 53°
s = 9.5836

c. Solve for a in the figure below:



Solution: ∠B is an acute angle, b is the opposite side, and a is the side adjacent to ∠B. Use TOA, that is

$$\theta = \frac{opposite}{adjacent} = \frac{b}{a}$$
$$B = \frac{10.6}{a}$$
$$tan\ 67° = \frac{10.6}{a}$$
$$a = \frac{10.6}{tan\ 67°}$$

By using scientific calculator, key in 10.6 divided by tan 67°
a = 4.4994
$a \approx 4.5$ (a is approximately 4.5)

d. Solve for c in figure above.
Solution: ∠B is an acute angle, b is the opposite side and c is the hypotenuse of the given acute angle.
Use SOH, that is

$$sin\ \theta = \frac{opposite}{hypotenuse}$$

$$B = \frac{b}{c}$$

$$sin\ 67° = \frac{10.6}{c}$$

$$c\ sin\ 67° = 10.6$$

$$c\ sin\ 67° = 10.6$$

$$c = \frac{10.6}{sin\ 67°}$$

By using scientific calculator, key in 10.6 divided by sin 67°

$$c = 11.5154$$

$c \approx 11.52$ (c is approximately 11. 52)

Example 2: Using the Calculator to Find Trigonometric Ratios

As presented in the previous sections, the values of the trigonometric ratios for any angle are constant, regardless of the length of the sides. These values can also be found using a calculator. Also, you can use a calculator to find an angle when you are given a trigonometric ratio.

a. Finding a ratio given the angle
Example 3:
To find the value of sin 38°, ensure that your calculator is operating in degrees.

Solution:
Press sin 38 = 0.615661475
The calculator should give sin 38° = 0.616, correct to three decimal places.
b. Finding an angle given the ratio
In finding the size of the angle to the nearest minute, given the value of the trigonometric ratio, just follow the steps in the examples below.

Example 4: sin θ = 0.725, find θ to the nearest minute

Solution: Press 2ndF sin 0.725 = 46.46884783

To convert this to degrees/minutes/seconds mode,
Press 2ndF D°M'S

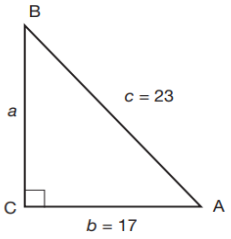
The calculator gives you 46° 28' (nearest minute)

c. Degrees and minutes
So far, all angles have been in whole degrees. However, one degree can be divided equally into 60 minutes. Further, one minute can be divided equally into 60 seconds. Angle measurement can also be expressed in degree/minutes form. We can use a calculator to convert a degree measure from decimal form to degree, minute, and second form.

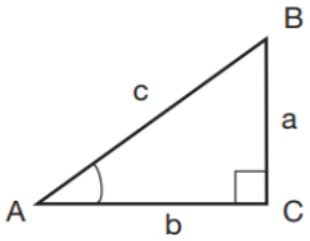
Example 5: Write 54.46° in degree and minutes, giving an answer correct to the nearest minute.

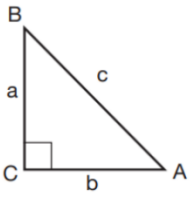
Solution: Press 54.46° 2ndF D°M'S

The calculator gives 54°27'36", or 54°28'. (nearest minute)

	In solving right triangles, it means finding the remaining parts. The following are the conditions for the given parts of right triangles: Example 6: Triangle BCA is right-angled at C. If c = 23 and b = 17, find ∠A, ∠B and a. Express your answers up to two decimal places. The is solving a right triangle given the measure of the two parts: the length of the hypotenuse and the length of one leg. *Solution*: Sketch the figure: a. Side b is the adjacent side of ∠A; c is the hypotenuse of right triangle BCA. Use CAH, that is $cos\ cos\ \theta = \frac{adjacent}{hypotenuse}$  $cos\ A = \frac{b}{c}$ $cos\ A = \frac{17}{23}$ $cos\ A = 0.7391$ Use scientific calculator to find an angle whose cosine value is 0.7391. Using a scientific calculator, A = 42.340° b. Since in part (a), it was already found that ∠A = 42.34°, then ∠B = 90° – 42.34° ∠B = 47.66° c. Using the Pythagorean theorem: $a^2 + b^2 = c^2$ $a^2 + (17)^2 = (23)^2$ $a^2 + 289 = 529$ $a^2 = 529 - 289$ $a^2 = 240$ $a = \sqrt{240}$ a = 15.49
Other Resources: *(Digital Modules and Textbook, Video Lessons)*	Textbooks and Modules: Mathematics Learner Material, Grade 9 Bacani, J., et al. Pre-calculus Book Grade 11. Sunshine Interlinks Publishing House, Inc., 2016. Alferez, Merle. MSA Trigonometry. MSA Academic Advancement Institute, 2001. Weimerskirch, M., & Pride, S. (Eds.). (2022). Trigonometry. University of Minnesota Board of Regents. https://open.lib.umn.edu/trigonometry/ Video Lessons: Solving Right Triangles | SOH CAH TOA | Trigonometry: https://www.youtube.com/watch?v=ljxkJHplMZU Solving Right Triangles - Missing Angle and Side of Right Triangle: https://www.youtube.com/watch?v=n4BEwESmzQw Solving Word Problems Involving Right Triangles: https://www.youtube.com/watch?v=AyiRdl38aT4 Angles of Elevation and Depression Practice: https://study.com/academy/lesson/video/angles-of-elevation-depression-practice-problems.html

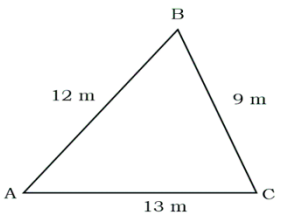
I. Activity 1	**Practicing Learned Skills**
II. Materials Needed:	pen and paper scientific calculator ruler, if a sketch is needed
III. Instructions:	*Perform the indicated tasks. Write your answer on a sheet of paper.* A. Use your calculator to find the value of the following, correct to two decimal places. 1. cos 85° 2. sin 7° 3. cos 65° 4. tan 35° 5. tan 63 ° B. Find the size of the angle θ (to the nearest degree) where θ is acute. 1. sin θ = 0.529 2. cos θ = 0.493 3. tan θ = 1.98 4. cos θ = 0.401 5. tan θ = 2.5 C. Solve the following problems. 1. Triangle BCA is right-angled at C if c = 27 and ∠A = 58°, find ∠B, b, and a. 2. Solve the remaining sides and angles of the right triangle ACB and right angled at C. a. a = 5cm c = 8 cm b. A = 540 b = 3 cm Reminder: • Check that the calculator is in degree mode. • Draw or visualize the right triangle before solving. • Label the hypotenuse, opposite side, and adjacent side correctly. • Round final answers as instructed.
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. How do the values of sine, cosine, and tangent change as the angle increases, and when is it most appropriate to use each trigonometric ratio? 2. Why is it important to correctly label the sides of a right triangle, round answers consistently, and use a calculator to verify manual trigonometric computations? 3. How do trigonometric ratios help in finding missing sides and angles of a right triangle, and how can these methods be applied to real-life measurement problems such as determining the height of a tree or building?

I. Activity 2	**Assessing Learning Outcomes**
II. Materials Needed:	pen and paper scientific calculator
III. Instructions:	*Read and understand the following problems. Solve each item.* A. Using the figure below, write the expressions that give the required unknown value.  1. If $A = 15°$ *and* $c = 37$, find a. 2. If $A = 76°$ *and* $a = 13$, find b. 3. If $A = 49°$ *and* $a = 10$, find c. 4. If $a = 21.2$ *and* $A = 71°$, find b. 5. If $a = 13$ *and* $B = 16°$, find c. 6. If $A = 19°$ *and* $a = 11$, find c. 7. If $c = 16$ *and* $a = 7$, find b.

	8. If $b = 10$ *and* $c = 20$, find a. 9. If $a = 7$ *and* $b = 12$, find A. 10. If $a = 8$ *and* $c = 12$, find B. B. Use the given figure to solve the remaining parts of right triangle ACB  General Mathematics Quarter 3 2 1. $b = 17\ cm$ *and* $c = 23\ cm$ 2. $c = 16$ *and* $a = 7$ 3. $b = 10$ *and* $c = 20$ 4. $b = 6$ *and* $c = 13$ 5. $c = 13$ *and* $a = 12$ C. Solve the following problems: 1. A kite string is 20 meters long and makes an angle of 40° with the ground. How high is the kite above the ground? 2. A ladder 8 meters long leans against a wall making an angle of 65° with the ground. How high up the wall does the ladder reach? 3. A wheelchair ramp is 5 meters long and rises 1.25 meters high. Find the angle of inclination of the ramp with the ground.
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. Have you completed all the tasks correctly? 2. Which items did you find easy, and which did you find challenging? 3. What common errors did you make? 4. How will you address your misconceptions?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials Needed:	pen and paper calculator
III. Instructions:	*Read and solve each problem.* 1. From a point on the ground, the angle of elevation to the top of a tree is 35°. If you are standing 15 meters from the base of the tree, how tall is the tree? 2. A 10-meter-tall lamp post casts a shadow 6 meters long on the ground. Find the angle of elevation of the sun. 3. A ladder is leaning against a wall making an angle of 60° with the ground. If the ladder is 5 meters long, how high on the wall does it reach? 4. A wheelchair ramp rises 1.2 meters vertically over a 5-meter length. Find the angle of inclination of the ramp with the ground. 5. A kite is flying at the end of a 40-meter string. If the string makes an angle of 50° with the ground, find how high the kite is above the ground. 6. A surveyor stands 120 m from the base of a hill. The angle of elevation to the top is 28°, and a tree halfway up the hill makes an angle of 15° with the horizontal. Find the height of the hill. 7. An observation tower stands on flat ground. From a point 50 m away, the angle of elevation to the top is 40°. From another point 30 m closer to the tower, the angle of elevation is 60°. Find the height of the tower. 8. A flagpole is mounted on top of a building. From a point 100 m from the base of the building, the angle of elevation to the top of the building is 30°, and the angle to the top of the flagpole is 35°. Find the height of the flagpole. 9. A person wants to cross a river and sees a point directly across. They measure the angles of elevation to a hill on the opposite bank from two points 40 m apart

	along their side of the river: 25° from the first point and 35° from the second. Find the height of the hill. 10. A cable car travels up a mountain along a straight cable 1.2 km long, making an angle of 22° with the horizontal. If the cable car station is at the base, find: a. the vertical height gained b. the horizontal distance covered **The** **Task A: Identify the Ratio** Determine whether sine, cosine, or tangent should be used in each situation. 1. Finding the height of a building using distance from the building and angle of elevation 2. Finding the length of a ladder leaning against a wall 3. Finding the angle formed by a ramp and the ground **Task B: Complete the Table** **Given Sides** | **Trigonometric Ratio** Opposite and Hypotenuse | Adjacent and Hypotenuse | Opposite and Adjacent | **Task C: Digital Practice** Using Desmos Graphing Calculator: • Encode the trigonometric equations. • Verify your answers using the calculator. Example: $\tan 35° = h15$ **For Enhancement** 1. A surveyor stands 120 m from the base of a hill. The angle of elevation to the top is 28°, and a tree halfway up the hill makes an angle of 15° with the horizontal. Find the height of the hill. 2. An observation tower stands on flat ground. From a point 50 m away, the angle of elevation to the top is 40°. From another point 30 m closer to the tower, the angle of elevation is 60°. Find the height of the tower. 3. A flagpole is mounted on top of a building. From a point 100 m from the base of the building, the angle of elevation to the top of the building is 30°, and the angle to the top of the flagpole is 35°. Find the height of the flagpole. 4. A person wants to cross a river and sees a point directly across. They measure the angles of elevation to a hill on the opposite bank from two points 40 m apart along their side of the river: 25° from the first point and 35° from the second. Find the height of the hill. 5. A cable car travels up a mountain along a straight cable 1.2 km long, making an angle of 22° with the horizontal. Find: a. the vertical height gained b. the horizontal distance covered
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. Which activity (problem-solving, remediation tasks, or enhancement tasks) helped you understand trigonometric ratios and their real-life applications the most? Explain why. 2. What challenges did you encounter while solving the trigonometric problems, and what support or teaching strategies would have helped you overcome those difficulties? 3. How effective were the learning resources and activities (e.g., real-life problems, Desmos Graphing Calculator, remediation tasks, and enhancement activities) in helping you learn? What suggestions do you have to improve future lessons?

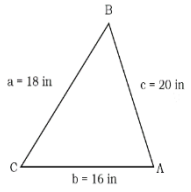
Lesson 8.2. Area of Oblique Triangles	
Objectives:	At the end of the lesson, learners should be able to: 1. illustrate an oblique triangle. 2. compute area of oblique triangles using Heron's Formula and the rule Area = $\frac{1}{2}$absinC 3. solve problems involving the area of oblique triangles.
Key Ideas and Example	**Finding the Area of Non-right Triangle** **Oblique triangle -** An oblique triangle is simply a triangle without a right angle. It may be acute (all angles < 90°) or obtuse (one angle > 90°). **The Heron's Formula (SSS)** Finding the Area of a Triangle Using Heron's Formula When all three sides of a triangle are known and no angle is given, we can find its area using Heron's Formula. This method does not require the height or any angles — only the side lengths. The formula is: $A=s(s-a)(s-b)(s-c)$ where: • A= area of the triangle • a,b,c= lengths of the three sides • s= semiperimeter of the triangle, calculated as $s=a+b+c2$ **The Trigonometric Rule** When a triangle is not a right triangle, its height is not easily measured. However, if two sides and the included angle are known, we can find its area using a trigonometric rule. The formula is: $A=12absin$C where: • A= area of the triangle • a and b= lengths of the two sides • C= included angle between sides aand b Example 1: Mang Lito's farm plot has sides measuring 12 meters, 9 meters, and 13 meters. What is the area of his plot?  Solutions: 1. Given triangle sides: a = 9m, b = 13 m, and c =12 m, we can illustrate the triangle plot. 2. Calculate semi-perimeter (s): $s= a+b+c2$ $s= 9+13+122$ $s= 342$ $s=17$ m

	3. Apply Heron's Formula: $A = s(s-a)(s-b)(s-c)$ $A = 17(17-9)(17-13)(17-12)$ $A = 17(8)(4)(5)$ $A = 2720$ $A = 2720$ $A \approx 52.15 \text{ m}^2$ Therefore, the area of △ABC is about 52.15 square meters. Example 2: Mang Pedro is building a triangular roof beam. He measures two sides: • Side a = 15 meters • Side b = 20 meters The angle between these two sides is 70°. Question: Find the area of the triangular roof beam. Solutions: Given: a = 15 m, b = 20 m, $C = 70^0$ $A = 12ab\sin C$ $A = 121520\sin(700)$ $A = 150\sin(700)$ $A \approx 140.96 \text{ m}^2$ Therefore, the area of the triangle is approximately 140.96 square meters.
Other Resources: *(Digital Modules and Textbook, Video Lesson)*	Bacani, J., et. al. Pre-calculus book Grade 11. Sunshine Interlinks Publishing House, Inc. 2016 Alferez, Merle. MSA Trigonometry, MSA Academic Advancement Institute, 2001. Weimerskirch, M., & Pride, S. (Eds.). (2022). Trigonometry. University of Minnesota Board of Regents. https://open.lib.umn.edu/trigonometry/ Moyer, E. Schaum's Outline of Trigonometry. McGraw Hill, 2018 Peterson, D., *Solving an Oblique Triangle Part I.* (2020, March) Retrieved May 26, 2026, from https://www.themathdoctors.org/solving-an-oblique-triangle-part-i/ OpenStax. *Algebra and Trigonometry* (2012). Retrieved May 26, 2026, from https://openstax.org/books/algebra-and-trigonometry/pages/10-key-concepts Oblique Triangle Application. (2024, March) Retrieved May 26, 2026, from https://www.cliffsnotes.com/study-notes/6258008 Hughes,B. et. al, CK-12 Trigonometry Concepts (2026, May 02) Retrieved May 26, 2026 from https://flexbooks.ck12.org/cbook/ck-12-trigonometry-concepts/section/5.5/primary/lesson/herons-formula-trig/ https://www.youtube.com/watch?v=TfrUy3fbgII https://www.youtube.com/watch?v=vPt6j5e_uhE https://www.youtube.com/watch?v=J8f18nWaIHE

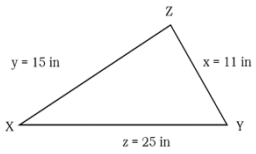
I. Activity 1:	**Practicing Learned Skills**
II. Materials Needed:	pen and paper
III. Instructions:	*Solve the following problems individually. Show complete step-by-step solutions. Afterward, pair-check with your seatmate, then volunteer to explain one problem on the board.* 1. A triangular fish pen in Laguna de Bay has sides 16 m, 18 m, and 25 m. Find its area. 2. During typhoon season, a barangay is constructing a triangular roof for an evacuation center. The roof beams measure a = 20 m, b = 25 m, with an included angle of 40°. Find the roof's triangular area. 3. A triangular rice field in Isabela has sides 40 m, 55 m, and 70 m. Find its area. Round to two decimal places.

IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. What challenges did you encounter while solving the problems, and how did you overcome them? 2. How are the concepts of finding the area of oblique triangles applied in real-life situations such as construction, farming, and community planning?

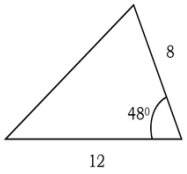
I. Activity 2:	**Assessing Learning Outcomes**
II. Materials Needed:	paper and pen
III. Instructions:	*Read each part of the activity carefully before answering. Make sure you understand the directions and what is being asked.* Part I. Concept Recall 1. Define an oblique triangle and give one real-life example in your community. 2. Explain briefly when to use Heron's Formula and when to use the trigonometric rule $\text{Area}=\frac{1}{2}ab\sin C$. 3. Complete the sentence: "We use Heron's Formula when _____, and we use $\text{Area}=\frac{1}{2}ab\sin C$ when _____." Part II. Solve the following problems completely. Show step-by-step solutions and include units. 1. The DPWH is surveying a triangular lot needed for a road widening project in Quezon City. The lot has sides measuring 35 m, 40 m, and 50 m. Find its area so they know how much land to acquire. 2. After a powerful typhoon in Albay, a family is rebuilding their home with stronger triangular roof supports to withstand future storms. Two support beams measure 18 meters and 22 meters, forming an included angle of 42°. Using trigonometry, calculate the area of the triangular roof support. 3. The city government of Baguio is designing a small triangular park with sides 20 m, 26 m, and 30 m. Find the park's area so they can estimate grass and landscaping costs. 4. A company in Metro Manila is installing a triangular solar-powered billboard. Two sides measure 14 m and 19 m, with an included angle of 37°. Find its area. If each square meter costs ₱1,200 to install, how much is the total cost? Part III. Read and analyze each problem below and answer the questions. 1. A student solved for the area of a triangle with sides 20 m, 25 m, and 30 m using $\text{Area}=\frac{1}{2}ab\sin C$. Is the formula used correct? Why? If it's incorrect, what formula should have been used? 2. If you only know two sides of a triangle and a non-included angle, can you directly apply Heron's Formula or $\text{Area}=\frac{1}{2}ab\sin C$? Justify your answer.
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.*

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials Needed:	pen and paper calculator
III. Instructions:	**For Remediation** *Solve the following problems. Show your complete solutions.* 1. Determine the area of triangle ΔABC shown in the figure. Round your answer to the nearest hundredths. 

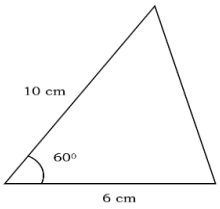
2. Calculate the area bounded by ΔXYZ in the given figure below. Round your answer to the nearest hundredths.



3.What is the area of the region bounded by the triangle on the right? Round your answer to the nearest hundredths.



4. Find the area of the triangle below.



5. Mang Edong is measuring a triangular section of land for irrigation. Two sides of the plot are 30 m and 25 m, with an included angle of 45°. Find the area of the triangular section.

For Enhancement

Solve each problem.

1. A triangle has sides b=10 m and c=13 m, with included angle A=55°.
   - Sketch the triangle:
   - Draw side b = 10 m.
   - At one end, use a protractor to draw a 55° angle.
   - From that angle, draw side c = 13 m.
   - Connect the endpoints to complete the triangle.
   - Label the triangle with sides and angle.
   - Calculate the area.
2. A triangle has sides a=8 ft. and c=11 ft., with included angle 70°.
   - Sketch the triangle**:**
   - Draw side a = 8 m.
   - At one end, use a protractor to draw a 70° angle.
   - From that angle, draw side c = 11 m.
   - Connect the endpoints to complete the triangle.
   - Label the triangle with sides and angle.
   - Calculate the area.

IV. Reflection:

To help you reflect on your learning experience, answer the following feedback and reflection questions

1. What difficulties did you encounter while solving the problems or sketching the triangles, and how could the teacher better support your learning in future lessons?
2. How effective were the examples, practice problems, and enhancement tasks in improving your understanding and confidence in solving triangle area problems? What suggestions do you have to make the lesson more engaging or easier to understand?

Lesson 8.3 Triangles and Navigation	
Objectives:	By the end of this module, you will be able to: 1. apply the Law of Sines and the Law of Cosines to calculate unknown sides and angles in non-right triangles; 2. calculate unknown heights and distances using angles of elevation and depression; and 3. use true and compass bearings to map positions and directions in real-world scenarios.
Key Ideas and Examples:	**Worded Problems in Trigonometry** **A. Applying the Law of Sines and Law of Cosines** *Problem 1*: A surveyor is measuring a triangular piece of land. Two points on the boundary are 10 meters apart from a third point, and the angle between the two measured sides is 30°. If one side measures 10 meters and the opposite angle is 30°, determine the length of the side opposite a 45° angle. *Solution*: Using the Law of Sines: $\frac{10}{\sin 30°} = \frac{x}{\sin 45°}$ $x = \frac{10 \sin 45°}{\sin 30°}$ x ≈ 14.14 m *Answer*: The unknown side is approximately 14.14 meters. *Problem 2*: Two roads meet at an intersection and form an angle of 60°. A delivery truck travels 8 km along one road, while another truck travels 12 km along the other road. How far apart are the two trucks? *Solution*: Using the Law of Cosines: $c^2 = a^2 + b^2 - 2ab \cos C$ $c^2 = 8^2 + 12^2 - 2(8)(12) \cos 60°$ $c^2 = 64 + 144 - 192(0.5)$ $c^2 = 112$ $c = \sqrt{112}$ c ≈ 10.58 km Answer: The trucks are approximately 10.58 km apart. *Problem 3:* A rescue team needs to locate a stranded hiker. From the rescue station, the hiker is observed at an angle of 40° from a reference direction. A second observation point is 65° from the same reference direction. If the distance from the station to the first observation point is 15 km, determine the remaining sides of the triangular search area. *Solution*: First, determine the third angle: C = 180° − 40° − 65° C = 75° Using the Law of Sines: $\frac{15}{\sin 40°} = \frac{x}{\sin 65°}$ $x = \frac{10 \sin 65°}{\sin 40°}$ x ≈ 21.52 km

For the other side:
$\frac{15}{\sin 40°} = \frac{y}{\sin 75°}$
$y = \frac{15 \sin 75°}{\sin 0°}$
y ≈ 22.40 km

Answer: The remaining sides are approximately 21.52 km and 22.40 km.

B. Heights and Distances Using Angles of Elevation and Depression
Problem 4: Maria is standing 20 meters away from the base of a flagpole. She looks up at the top of the flagpole and measures an angle of elevation of 30°. Assuming Maria's eye level is at the ground, how tall is the flagpole?

Solution:
Using tangent:
$\tan 30° = \frac{h}{20}$
h = 20 tan 30°
h = 20(0.5774)
h ≈ 11.55 m

Answer: The flagpole is approximately 11.55 meters high.

Problem 5: A student standing on the ground observes the top of a school building at an angle of elevation of 45°. The student is 35 meters away from the building. How high is the building?

Solution:
$\tan 45° = \frac{h}{35}$
Since tan 45° = 1:
h = 35

Answer: The school building is 35 meters high.

Problem 6: A surveyor wants to determine the height of a communication tower. From one location, the angle of elevation to the top of the tower is 35°. The surveyor then moves 20 meters closer to the tower, and the angle of elevation becomes 50°. How high is the tower?

Solution:
Let x represent the distance from the second observation point to the tower.
From the second position:
h = x tan 50°
From the first position:
h = (x + 20) tan 35°
Equating:
x tan 50° = (x + 20) tan 35°
1.1918x = 0.7002x + 14.004
0.4916x = 14.004
x ≈ 28.49
Now find the height:
h = 28.49 tan 50°
h ≈ 33.96 m

Answer: The communication tower is approximately 33.96 meters high.

	C. True and Compass Bearings *Problem 7*: A fishing boat leaves the shore and travels 20 kilometers on a true bearing of 090°. In what direction does the boat travel, and where is it located relative to the shore? *Solution*: A bearing of 090° indicates due east. Therefore, the boat travels: 20 km east *Answer*: The boat is 20 kilometers east of the shore. *Problem 8*: A rescue team travels 15 kilometers from its base camp on a true bearing of 060°. How far north and how far east of the base camp is the team? Solution: For a bearing of 060°: N = 15 cos 60° N = 7.5 km Eastward distance: E = 15 sin 60° E ≈ 12.99 km Answer: The rescue team is approximately 7.5 km north and 12.99 km east of the base camp. *Problem 9*: A cargo ship leaves a port and travels 40 kilometers on a true bearing of 030°. It then changes direction and travels another 30 kilometers on a true bearing of 120°. Determine the ship's approximate distance and true bearing from the port. Solution: For the first journey: $N_1 = 40 \cos 30° \approx 34.64$ km $E_1 = 40 \sin 30° = 20$ km For the second journey: $N_2 = 30 \cos 120° = -15$ km $E_2 = 30 \sin 120° \approx 25.98$ km Total northward displacement: N = 34.64 − 15 = 19.64 km Total eastward displacement: E = 20 + 25.98 = 45.98 km Resultant distance: $d = \sqrt{(19.64^2 + 45.98^2)}$ d ≈ 50.00 km Determine the bearing: $\theta = \tan^{-1}(45.98/19.64)$ θ ≈ 66.9° Thus, the true bearing is approximately 067°. Answer: The ship is approximately 50 km from the port on a true bearing of 067°.
Other Resources: *(Digital Modules and Textbook, Video Lesson)*	Bacani, J., et. al. Pre-calculus book Grade 11. Sunshine Interlinks Publishing House, Inc. 2016 Alferez, Merle. MSA Trigonometry, MSA Academic Advancement Institute, 2001. Weimerskirch, M., & Pride, S. (Eds.). (2022). Trigonometry. University of Minnesota Board of Regents. https://open.lib.umn.edu/trigonometry/ Moyer, E. Schaum's Outline of Trigonometry. McGraw Hill, 2018 Peterson, D., *Solving an Oblique Triangle Part I.* (2020, March) Retrieved May 26, 2026, from https://www.themathdoctors.org/solving-an-oblique-triangle-part-i/

	OpenStax. *Algebra and Trigonometry* (2012). Retrieved May 26, 2026, from https://openstax.org/books/algebra-and-trigonometry/pages/10-key-concepts Oblique Triangle Application. (2024, March) Retrieved May 26, 2026, from https://www.cliffsnotes.com/study-notes/6258008 Hughes,B. et. al, CK-12 Trigonometry Concepts (2026, May 02) Retrieved May 26, 2026 from https://flexbooks.ck12.org/cbook/ck-12-trigonometry-concepts/section/5.5/primary/lesson/herons-formula-trig/

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen and paper
III. Instructions:	*Work collaboratively in groups of 3–4 members to solve the given problem. Record your group's final answers and be prepared to present and explain your solution during the class discussion. Listen to the teacher's feedback, compare your answers with the correct solution, and reflect on what you learned from the activity.* 1. A 6-meter ladder leans against a wall. The base is 4 meters from the wall. Find the height reached. 2. A student is 30 meters from a tower. The angle of elevation to the top is 40°. Find the height of the tower. 3. A lighthouse is 50 meters high. The angle of depression to a boat is 25°. Find the distance of the boat from the base. 4. A boat travels N 60° E for 80 km. Draw the direction and describe its movement. 5. Two sides of a triangle are 70 m and 50 m with an included angle of 60°. Find the third side using Law of Cosines.
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. How did you decide which formula or trigonometric ratio to use in each problem? 2. What helped your group correctly draw and label the diagram before solving the problem? 3. Which type of problem was easier for you: right triangle, elevation/depression, bearings, or oblique triangles? Why? 4. What common mistake should be avoided when solving problems involving trigonometric ratios or triangle laws? 5. How can the skills you practiced today be useful in real-life situations such as construction, navigation, or surveying?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper calculator
III. Instructions:	Part I. *Choose the letter of the correct answer.* 1. Which trigonometric ratio is defined as opposite over hypotenuse? A. Cosine B. Sine C. Tangent D. Cotangent 2. A triangle with one right angle is called: A. Acute triangle B. Oblique triangle C. Right triangle D. Obtuse triangle 3. What is an angle of elevation? A. Angle measured downward from horizontal B. Angle between two sides of a triangle C. Angle measured upward from horizontal D. Angle inside a triangle

	4. Which law is used to solve oblique triangles when all three sides are given? A. Law of Sines B. Law of Cosines C. Pythagorean Theorem D. Tangent Law 5. N 45° E is an example of: A. Angle of elevation B. Bearing C. Trigonometric ratio D. Depression angle Part II. *Solve each problem.* 1. A student is 25 meters away from a tower. The angle of elevation is 40°. Find the height of the tower. 2. A 10-meter ladder is placed 6 meters from a wall. Find the height it reaches. 3. A lighthouse is 60 meters high. The angle of depression to a boat is 30°. Find the distance of the boat from the base. 4. A plane travels 100 km on a bearing of N 60° E. 5. Two sides of a triangle are 70 m and 50 m with an included angle of 60°. Find the third side.
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. Which trigonometric ratio or theorem did you use most often in solving the problems? Why was it the appropriate choice for those situations? 2. How did you determine whether to use the Pythagorean Theorem, trigonometric ratios, or the Law of Cosines in each problem? 3. Which problem did you find the most challenging? What difficulties did you encounter, and how were you able to solve or overcome them? 4. How are angles of elevation, angles of depression, and bearings useful in real-life professions such as engineering, surveying, aviation, architecture, or navigation? Give one specific example. 5. If you were asked to design a simple structure or plan a safe route using trigonometry, how would the concepts learned in today's lesson help you make accurate decisions? Explain your answer.

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials:	pen and paper calculator
III. Instructions:	**For Remediation** *Read each problem carefully. Identify the given information and draw a simple diagram if needed. Choose the appropriate trigonometric ratio or theorem, show your complete solution step by step, and write your final answer with the correct unit. If you need assistance, work with your teacher or a learning partner and use the guided examples discussed in class as your reference.* 1. A 5-meter ladder is 3 meters from the wall. Find the height reached. 2. A student is 10 meters from a tree. The angle of elevation is 30°. Find height. **For Enhancement** *Read the problem carefully. Solve each task using the appropriate trigonometric concepts, show your complete solutions, draw a labeled diagram, and provide your final answers and interpretation.*

	Problem: A drone is flying at a height of 50 meters above the ground. A student observes the drone at an angle of elevation of 40°. At the same time, the drone moves on a bearing of N 30° E for 100 meters.
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. What information was given in each problem, what quantity were you asked to find, and how did drawing and labeling the triangle help you choose the appropriate trigonometric ratio or theorem to solve it? 2. What challenges did you encounter while solving the problems, how did you overcome them, and how can the concepts of right triangles and angles of elevation be applied to real-life situations such as using a ladder or measuring the height of a building or tree?

Unit 9. Measurement and Geometry in Real-Life Applications			
Name:		**Grade & Section:**	

Lesson 9.1. Estimating and Measuring Lengths and Distances	
Objectives:	At the end of the lesson, you should be able to: 1. estimate at least five classroom objects within a reasonable margin of error; 2. measure selected objects using appropriate tools and correct unit;, and 3. explain at least two practical uses of estimation and measurement based on the activity outputs.
Key Ideas and Example:	**Estimation** Estimation is making a reasonable guess or approximation of a measurement without using measuring tools. Examples: • Estimating the height of a classroom door • Estimating the width of a table • Estimating the distance from the classroom to the flagpole Measurement is the process of determining the exact size, length, height, width, or distance of an object using standard measuring tools. Common Units of Length • Millimeter (mm) • Centimeter (cm) • Meter (m) • Kilometer (km) Common Measuring Tools • Ruler • Meter Stick • Tape Measure • Trundle Wheel Why Estimate First? • Saves time • Help make quick decisions Provides a basis for comparison before actual measurement **Example 1. Measuring a Notebook** Maria estimates that her notebook is **25 cm** long. Using a ruler, she measures its actual length and finds that it is **23 cm**. Difference: Difference = |25 – 23| Difference = 2 cm **Therefore, Maria's estimate differs from the actual measurement by 2 cm.** **Example 2. Measuring a Classroom Door** A learner estimates the height of a classroom door as **2.2 m**. Using a tape measure, the actual height is **2.0 m**. Difference: Difference = |2.2 – 2.0| Difference = 0.2 m **Therefore, the estimate is 0.2 m greater than the actual measurement.** **Example 3. Distance to the Flagpole** A student estimates the distance from the classroom to the flagpole as **30 m**. Using a tape measure, the actual distance is **28 m**. Difference: Difference = |30 – 28| Difference = 2 m **Therefore, the estimate differs from the actual distance by 2 m.**

Other Resources: (*Digital Modules and Textbook, Video Lesson)*	Century Tech. “Solving Problems with Volume and Capacity | Mathematics.” YouTube video, 2023. Accessed June 10, 2026. YouTube SmileRoad. “LVII P1 Volume and Capacity in Cost Estimation | GR11 SHS | Gen Math | Q3 MG.” YouTube video, 2025. Accessed June 10, 2026. YouTube Unknown. “Finding the Perimeter and Area of a Room.” YouTube video. Accessed June 10, 2026. YouTube

I. Activity 1:	**Practicing Learned Skills**
II. Materials Needed:	pen and paper ruler or tape measure
III. Instructions:	*Estimate the measurements of the listed objects without using measuring tools, then measure each object using the appropriate tool. Compare your estimate with the actual measurement by finding the difference between the two.* **Activity Table** **Object** | **Estimated Measurement** | **Actual Measurement** | **Difference** Notebook Chair Teacher's Table Door Blackboard **Questions** 1. Which estimate was closest to the actual measurement? 2. Which object was most difficult to estimate? 3. Which measuring tool was most useful? 4. Why is estimation important before measuring? 5. How can accurate measurement help in everyday life?
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. What did you learn about the relationship between estimation and actual measurement? 2. How can these skills help you in school, at home, and in future careers?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials Needed:	pen and paper
III. Instructions:	*Read each question carefully and write the CORRECT answer.* 1. Which of the following best describes estimation? A. Finding an exact measurement B. Making an approximate judgment of a value C. Using a ruler to measure length D. Converting units of measurement 2. Which tool is most appropriate for measuring the length of a classroom? A. Protractor B. Thermometer C. Tape Measure D. Compass 3. A learner estimated a table length as 1.5 m. The actual length is 1.4 m. What is the difference? A. 0.1 m B. 0.2 m C. 0.5 m D. 1.0 m

	4. Why is estimation useful? A. It replaces measurement. B. It provides quick approximation. C. It eliminates errors. D. It is always exact. 5. Why are standard measuring tools preferred over handspans and footsteps? A. They are more colorful. B. They are lighter. C. They are more accurate and consistent. D. They are easier to carry.
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. How well did the activities and questions help you understand estimation, measurement, and the importance of using standard measuring tools? 2. Which question or concept was most difficult for you, and what could the teacher do to help you understand it better? 3. How confident are you in applying estimation and measurement in real-life situations after the lesson? What suggestions can you give to improve the activities or assessment?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials Needed:	pen and paper calculator
III. Instructions:	*Solve the following problems. Show your complete solutions.* **For Remediation** 1. Mark estimates the width of the window, using arms, to be about 1.9 meters. At the shop, the material costs ₱120 per meter. a. How many meters of material should Carl buy? b. How much will the material total cost? 2. A carpenter estimates that the length of a wooden plank is about 1.7 meters. When measured accurately, it is 1.9 meters. a. How many meters of wood should he pay for if wood is sold only by the meter? b. If the price is ₱120 per meter, how much will he spend? **For Enrichment** A family's car odometer reads 48,625 km before a road trip. After reaching their destination, it reads 49,040 km. a. How many kilometers did the family travel? b. If the car consumes 1 liter of gasoline for every 14 km, how many liters of gasoline were used for the trip? c. If gasoline costs ₱68.50 per liter, how much did the family spend on fuel for the trip?
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions.* 1. How did the activities help you understand estimation, accurate measurement, rounding, and cost computation in real-life situations? 2. What part of the problems was most challenging for you, and what helped you understand or solve it? *3.* How effective were the activities in helping you apply mathematical skills to everyday situations? What could the teacher do to make similar activities easier to understand or more engaging?

Lesson 9.2. Calculating Area and Perimeter for Practical Purposes	
Objectives:	At the end of the lesson, you should be able to: 1. calculate the area and perimeter of rectangular, square, triangular, circular and composite residential spaces; 2. solve contextual problems involving room renovation and garden lay outing; and 3. explain in writing or orally at least two uses of area and perimeter in home improvement and budgeting tasks based on the performance activity.
Key Ideas and Examples	**Perimeter and Area** **Perimeter** The perimeter is the total distance around a closed figure. Formula for a Rectangle $P=2L+2W$ where: • P= Perimeter • L= Length • W= Width Formula for a Square $P=4s$ where: • s= length of one side **Area** The area is the amount of surface covered by a two-dimensional figure. Formula for a Rectangle $A=L\times W$ Formula for a Square $A=s2$ **Importance of Area and Perimeter** Area and perimeter are used in: • Flooring and tiling • Painting walls • Fencing land • Gardening • Construction and architecture • Interior design **Example 1. Finding the Perimeter of a Rectangle** A classroom bulletin board measures 2 m long and 1 m wide. $P=2L+2WP=2(2)+2(1)P=4+2P=6$ m **Answer:** The perimeter is **6 meters**. **Example 2. Finding the Area of a Rectangle** A study table measures 1.5 m long and 0.8 m wide. $A=L\times WA=1.5\times 0.8A=1.2$ m2 **Answer:** The area is **1.2 square meters**. **Example 3. Finding the Area and Perimeter of a Square** A floor tile has sides measuring 40 cm. **Perimeter** $P=4sP=4(40)P=160$ cm **Area** $A=s2A=402A=1600$ cm2 **Answer:** Area = 1600 cm²; Perimeter = 160 cm
Other Resources: *(Digital Modules and Textbook, Video Lesson)*	Century Tech. “Solving Problems with Volume and Capacity | Mathematics.” YouTube video, 2023. Accessed June 10, 2026. YouTube

	SmileRoad. “LVII P1 Volume and Capacity in Cost Estimation | GR11 SHS | Gen Math | Q3 MG.” YouTube video, 2025. Accessed June 10, 2026. YouTube Unknown. “Finding the Perimeter and Area of a Room.” YouTube video. Accessed June 10, 2026. YouTube

I. Activity 1:	**Practicing Learned Skills**
II. Materials Needed:	pen and paper meter stick or tape measure
III. Instructions:	Measure the dimensions of the following objects and compute their perimeter and area. **Activity Table** **Questions** 1. Which object has the largest perimeter? 2. Which object has the largest area? 3. Does a larger perimeter always mean a larger area? 4. What measuring tools did you use? 5. Why is accurate measurement important?
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the question below.* How can knowledge of area and perimeter help people in everyday life?

Object	Length	Width	Perimeter	Area
Notebook				
Book Cover				
Teacher's Table				
Whiteboard				
Window				

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials Needed:	pen and paper calculator
III. Instructions:	*Read each question carefully and write the CORRECT answer.* 1. What is the perimeter of a rectangle with length 8 m and width 4 m? A. 12 m B. 24 m C. 32 m D. 16 m 2. What is the area of a rectangle with length 10 m and width 5 m? A. 15 m^2 B. 30 m^2 C. 50 m^2 D. 60 m^2 3. A square has sides measuring 7 cm. What is its perimeter? A. 14 cm B. 21 cm C. 28 cm D. 49 cm 4. A square has sides measuring 7 cm. What is its area? A. 14 cm^2 B. 28 cm^2 C. 42 cm^2 D. 49 cm^2 5. Which situation requires finding the area? A. Putting a fence around a garden B. Framing a picture

	C. Determining the amount of floor tiles needed D. Measuring the edge of a table
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. How well did the activities and questions help you understand estimation, measurement, and the importance of using standard measuring tools? 2. Which question or concept was most difficult for you, and what could the teacher do to help you understand it better? 3. How confident are you in applying estimation and measurement in real-life situations after the lesson? What suggestions can you give to improve the activities or assessment?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials Needed:	pen and paper calculator
III. Instructions:	*Answer the following questions. Show your solutions for computation items and use neat sketches for drawing items.* 1. A rectangular living room has a length of 8 m and a width of 6 m. If tiles cost ₱300 per square meter, how much will it cost to tile the entire floor? 2. Mr. Cruz is renovating his backyard. He plans to build a rectangular patio that measures 10 meters long and 6 meters wide. At the center of the patio, he will install a circular garden with a radius of 2 meters. a. What is the area of the patio? b. What is the perimeter of the patio? c. What is the area of the circular garden? d. How much space (area) remains in the patio for tiles after placing the circular garden? e. If tiles cost ₱250 per square meter, how much will Mr. Cruz spend on tiling the remaining area of the patio? 3.Maria wants to install curtain rods along the four walls of her rectangular living room. The room measures 6 meters long and 4 meters wide. How many meters of curtain rods does Maria need? 4.A family plans to put grass on their rectangular garden that measures 10 meters long and 8 meters wide. What is the total area of the garden that will be covered with grass? 5.A rectangular kitchen has a length of 5 meters and a width of 3 meters. The family wants to put tiles on the floor, and each square meter of tile costs ₱400. How much will they spend for the tiles?
IV. Reflection:	*As a final synthesis of your learning, answer each of the question below.* 1. Explain the difference between area and perimeter. 2. Give one real-life situation where each is used.

Lesson 9.3. Volumes, Capacity, and Cost Estimation	
Objectives:	At the end of the lesson, learners should be able to: 1. distinguish between volume and capacity; 2. calculate the volume of various 3D shapes and convert between common units of volume and capacity to solve practical problems; and 3. apply their knowledge of volume and capacity to solve cost estimation problems by first calculating the required amount of a material and then determining the total cost based on the material's unit price.
Key Ideas and Examples:	**Understanding Volume, Capacity, and Cost Estimation** **Key Concepts** **Volume** refers to the amount of space occupied by a three-dimensional object. **Capacity** refers to how much a container can hold.

Common units used:
cubic centimeters (cm^3)
cubic meters (m^3)
liters (L)
milliliters (mL)

The following conversion relationships shall be used for estimation.

$$1000\ cm^3 = 1\ L$$
$$1\ m^3 = 1000\ L$$

Example 1. **Balik-Bayan Box**
A balik-bayan box measures **0.8 m × 0.6 m × 0.5 m** and will be filled with liquid soap sold at **₱40.00 per liter**. Find the volume and estimated total cost of liquid soap.



Solution:
Volume:

$$V = 0.8 \times 0.6 \times 0.5 = 0.24\ m^3$$

Convert cubic meters to liters:

$$0.24\ m^3 = 240\ L$$

Compute the cost:

$$240 \times 40 = 9600$$

Therefore, the estimated cost is **₱9,600**.

Example 2. **Cylindrical Water Tank**
A cylindrical water tank has a diameter of **2 m** and a height of **4 m**. Water costs **₱35.00 per kiloliter**. Find the volume and estimated total cost of water.



Solution:
The radius is half of the diameter:
Radius = 1 m
Volume of cylinder:
Substitute the given values:

$$V = \pi(1)^2(4)$$

Simplify:

$$V = 4\pi\ m^3$$

(Capacity) Convert to Kiloliters
Since:

$$1\ m^3 = 1\ kL$$

then:

$$4\pi\ m^3 = 4\pi\ kL$$

Compute the Cost

$$35(4\pi) = 140\pi$$

Thus, the total cost is:

$$140\pi \approx 439.82$$

Therefore, the estimated cost to fill the tank is approximately **₱440.00**.

Example 3. **Rice Container**
A rice container measures 40 cm × 30 cm × 50 cm. If the container is completely filled with rice and the density of rice is approximately 0.85 kg/L, and rice costs ₱55.00 per kilogram, determine the estimated cost of the rice.



Solution:
Volume of rectangular prism:

$$V = lwh$$

Substitute the values:

$$V = 40 \times 30 \times 50 = 60000\ cm^3$$

(Capacity) Convert to liters:

$$60000\ cm^3 = 60\ L$$

	Estimate the mass of rice: $60\ L \times 0.85\ kg/L = 51\ kg$ Compute the total cost: $51 \times 55 = 3300$ Therefore, the estimated cost is **₱2,805.00**. *Example 4.* **Estimating the Cost of Painting a Room** A rectangular room is 5 meters long, 4 m wide, and 3 m high. The four walls will be painted, but the door (2 m × 0.9 m) and two windows (1.5 m × 1 m each) will not be painted. One liter of paint can cover approximately 10 m^2, and paint costs **₱350.00 per liter**. **Determine the estimated cost of the paint needed to paint the room.** Solution 1. Find the total area of the four walls. Two walls measure 5 m × 3 m: $2(5)(3) = 30\ m^2$ Two walls measure 4 m × 3 m: $2(4)(3) = 24\ m^2$ Total wall area: $30 + 24 = 54\ m^2$ 2. Subtract the areas of the door and windows. Door: $2(0.9) = 1.8\ m^2$ Two windows: $2(1.5)(1) = 3\ m^2$ Area to be painted: $54 - 1.8 - 3 = 49.2\ m^2$ 3. Estimate the amount of paint needed. Since 1 liter covers 10 m^2: $49.2 \div 10 = 4.92\ L$ Because paint is purchased by the liter, round up to 5 liters. 4. Calculate the estimated cost. 5 × ₱350 = ₱1,750 Answer The estimated cost of paint is ₱1,750.00.
Other Resources: *(Digital Modules and Textbook, Video Lesson)*	https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa https://youtu.be/UHUx5ygYlCM?si=UhXVSftThwlwmVG-

I. Activity 1:	**Practicing Learned Skills**
II. Materials Needed:	pen and paper calculator
III. Instructions:	*Read each problem carefully, identify the given information and the appropriate formula, then solve for the required volume, surface area, capacity, or cost. Show all necessary formulas, computations, unit conversions, and final answers. Check your work for accuracy, and briefly explain the mathematical concepts and steps you used to arrive at your solution* Home Improvement Projects: 1. **Filling a Water Drum -** A cylindrical water drum has a radius of 0.3 m and a height of 1 m. If water costs ₱50 per cubic meter, how much will it cost to fill the drum completely?

	2. **Painting a Water Tank -** A cylindrical water tank has a radius of 0.8 m and height of 2 m. You need to paint the outside surface including the top but not the bottom. Paint costs ₱100 per square meter. Use Surface Area = $2\pi r^2 + 2\pi rh$. 3. **Constructing a Rectangular Fish Pond -** A rectangular pond is 4 m long, 3 m wide, and 1.2 m deep. It will be lined with cement 5 cm thick. Cement costs ₱2,000 per cubic meter. Compute the volume of cement needed and the total cost. School Facility Projects: 1. **Flower Box Volume -** A rectangular flower box measures 1.2 m long, 0.5 m wide, and 0.4 m deep. If soil costs ₱300 per cubic meter, find the total cost to fill one box. 2. **Rainwater Collector -** A cylindrical drum with a radius of 0.4 m and height of 1.2 m is used to collect rainwater. If the drum costs ₱250 per cubic meter of material, estimate the total cost. 3. **Cylindrical Water Tank with Dome -** A tank has a cylindrical base with radius 1.5 m and height 3 m, topped with a hemispherical dome. Material costs ₱400 per cubic meter. Find the total volume and cost of the tank. Business Projects: 1. **Ice Cream Container -** A cylindrical container has a radius of 7 cm and a height of 10 cm. If 1 liter of ice cream costs ₱250, find the cost to fill 10 containers. Remember: 1,000 cm^3 = 1 liter. 2. **Milk Carton Production -** A rectangular milk box measures 8 cm × 6 cm × 15 cm. If each carton costs ₱0.30 per 100 cm^3 to produce, find the cost of producing 20 cartons. 3. **Water Delivery Truck -** A cylindrical water tank has radius 0.6 m and length 3 m. It delivers water at ₱0.15 per liter. How much will the driver earn per trip if the tank is full? Remember: 1 m^3 = 1,000 L. Construction and Design Projects: 1. **Wooden Box Storage -** A rectangular wooden box measures 0.5 m × 0.4 m × 0.3 m. Wood costs ₱1,200 per cubic meter. Find the cost of wood to make one box. 2. **Concrete Pillars -** A house needs 4 cylindrical pillars, each with a radius of 0.25 m and a height of 3 m. Concrete costs ₱2,500 per cubic meter. Find the total concrete cost for all pillars. 3. **Rectangular Pool with Semi-Circular End -** A swimming pool is 8 m long, 4 m wide, and 1.5 m deep, with a semicircular end with radius 2 m. Water costs ₱35 per cubic meter. Find the total cost to fill the pool.
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. How did the real-life project problems help you understand and apply the concepts of volume, surface area, and cost estimation? 2. Which problem or type of project was most challenging for you, and what part of the solution did you find difficult? What could the teacher do to help you understand it better? 3. How effective were the project-based activities in helping you connect mathematics to real-life situations such as school facilities, business, and construction? What suggestions do you have for improving the activities?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials Needed:	pen and paper measuring tape, ruler, or meter stick calculator
III. Instructions:	*Estimate the measurements of the listed objects without using measuring tools, then measure each object using the appropriate tool. Compare your estimate with the actual measurement by finding the difference between the two.* 1. A cylindrical drum has a diameter of 1 m and a height of 1.5 m. a. Calculate its capacity in liters. b. If water costs ₱35 per kiloliter, estimate the total cost to fill the drum. 2. A cube has an edge of 20 cm. a. Find its volume in liters. b. If the cube will be filled with rice costing ₱50 per liter, calculate the total cost. 3. A water tank has a volume of 2 m^3. a. Determine how many liters of water it can hold. b. Explain why converting from cubic meters to liters is important when calculating water costs. 4. A box has a volume of 45,000 cm^3. a. Convert the volume to liters. b. If a product costs ₱40 per liter, determine the total cost to fill the box. 5. Which statement about cubic meters is correct? Show mathematically. a. 1 m^3 = 100 L b. 1 m^3 = 1,000 L c. 1 m^3 = 10,000 L
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. How did estimating, measuring, and comparing the measurements help you understand volume, capacity, and unit conversions? Which part of the activity was most challenging? 2. How did the real-life applications involving water, rice, and cost help you understand the importance of converting between cubic meters, cubic centimeters, and liters? What could the teacher improve to help you learn the lesson better?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials Needed:	pen and paper calculator
III. Instructions:	*Analyze and solve each problem* **Remediation Set** 1. A cylindrical water container has a diameter of 0.8 m and a height of 1 m. Find its capacity in liters. 2. A cube has an edge of 15 cm. Find its volume in liters. 3. A tank holds 1.2 m^3 of water. How many liters can it hold? 4. A box has a volume of 30,000 cm^3. Convert this to liters. 5. Which is correct: a. 1 m^3 = 500 L b. 1 m^3 = 1,000 L c. 1 m^3 = 5,000 L **Enhancement Set** 1. A cylindrical drum has a diameter of 1.2 m and a height of 2 m. Calculate its capacity in liters and estimate the cost to fill it if water costs ₱40 per kiloliter. 2. A cube-shaped container has an edge of 25 cm. Find its volume in liters and determine the total cost if rice costs ₱55 per liter.

	3. A water tank holds 3.5 m^3. Calculate how many liters it can hold and how much it would cost to fill it at ₱35 per kiloliter. 4. A rectangular box measures 0.7 m × 0.5 m × 0.6 m. Find its volume in liters and the total cost if a liquid product costs ₱45 per liter. 5. Which of these statements is correct, and explain why: a. 1 m^3 = 100 L b. 1 m^3 = 1,000 L c. 1 m^3 = 10,000 L
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions.* 1. How did the Remediation/ Enhancement activities help you understand more volume, capacity, and the conversion between cubic meters, cubic centimeters, and liters? 2. Which problem or step was most difficult for you, and what explanation, example, or support from the teacher would have helped you solve it more confidently? *3.* How effective were the real-life cost problems in helping you apply volume and capacity concepts? What would you suggest to improve the activities?

Lesson 10. Random Variables and Sampling			
Name:		**Grade & Section:**	

Lesson 10.1. Discrete Random Variables

Objectives:

At the end of the lesson, you are expected to:

1. define the concepts of random variables, discrete random variables, continuous random variables, expected value, variance, and standard deviation of a discrete random variable;
2. differentiate discrete random variables from continuous random variables based on their characteristics;
3. identify examples of real-life situations that can be modelled using discrete and continuous random variables; and
4. apply formulas to compute the expected value, variance, and standard deviation of a discrete random variable.

Key Ideas and Examples:

Random Variables

A **random variable** is a variable whose value is determined by chance. It may take numerical values depending on the outcome of the experiment. Random variables are usually denoted by capital letters such as X, Y, or Z, and they represent the set of possible outcomes of a random experiment.

Example 1:

A coin is tossed 3 times. Let X be the random variable denoting the number of heads. Find the values of the random variable X.

Solutions:

To accomplish this, first identify all possible outcomes of the given statistical experiment, and then determine the specific random variable defined in the problem, i.e.

Outcomes	TTT	TTH	THT	THH	HTT	HTH	HHT	HHH
x = no. of heads	0	1	1	2	1	2	2	3

Therefore, the possible values of the random variable X are 0, 1, 2 and 3.

Example 2:

The random variable Y denotes whether the outcome is even in a die roll (1 if even, 0 if odd). Find the possible values of Y.

Die Outcome	**Is it Even?**	Y = **Number of Even Numbers**
1	No	0
2	Yes	1
3	No	0
4	Yes	1
5	No	0
6	Yes	1

From the table, Y can take the values (0, 1).

More examples of random variables:

1. The number of hearts obtained when drawing five cards from a standard deck, the number of hearts drawn (0, 1, 2, 3, 4, or 5).
2. The number of heads or tails obtained when flipping a coin. (0,1)
3. The number obtained when rolling a die (1, 2, 3, 4, 5, or 6).
4. The number of heads that will appear in tossing a coin three times. (0, 1, 2, 3)
5. The sum of dots in rolling a pair of dice.

The importance of random variables lies in their ability to help us understand and predict uncertain real-world events. Each random variable has an underlying probability distribution, which specifies the likelihood of each possible value occurring. This allows us to estimate probabilities of different outcomes and make informed decisions based on this information.

Random variables are widely used in fields where risk assessment and decision-making are crucial, such as finance, engineering, and medicine. For example, in finance, random variables model investment returns, helping investors allocate resources wisely and manage risk effectively.

Types of Random Variables

A **discrete** random variable is a type of variable that takes values you can count.

Example 3:
A local agricultural trading hub inspects standard crates of native cardava bananas arriving from nearby farms. Let the discrete random variable X represent the exact number of under-ripe or bruised banana hands found per sample crate. Based on records from previous deliveries, the probability distribution is modeled as follows:

Number of Bruised Hands (X)	0	1	2	3
Probability P(X)	0.40	0.35	0.15	0.10

Other real-world examples include:

- The number of coins in your pocket.
- The number of learners in a classroom.
- The number of calls received at a help desk.
- The number of tricycle passengers picked up in a single trip.

A **continuous** random variable is one whose values are obtained through measurement.

Example 4:

1. The time it takes for a runner to finish a race.
2. The height of each learner in a classroom.
3. The exact amount of milk in a carton.
4. The temperature recorded at noon in a city.
5. The volume of water in a swimming pool.

Probability of a Discrete Random Variable

A **discrete random variable** is one that can take on countable values (e.g., 0, 1, 2, …). Each possible value has an associated probability, which satisfies:

1. $0 \leq P(X=x_i) \leq 1$ for each value x_i
2. $\sum_i P(X=x_i) = 1$

The probability distribution of a discrete random variable lists all possible values and their probabilities.

Graph of a Probability Distribution of a Discrete Random Variable

A **graph of a probability distribution** for a discrete random variable is a **probability histogram** or a **stick graph** (vertical line graph) that shows each possible value of the variable along with its probability.

Because the variable is *discrete*, the graph has **separate bars or lines**, not a continuous curve.

How to Graph a Discrete Probability Distribution

1. List the values of the random variable (e.g., 0, 1, 2, 3 heads).
2. List the probability of each value.
3. Draw the x-axis labeled with the values of the random variable.
4. Draw the y-axis labeled with "Probability".
5. Draw bars or vertical lines whose heights correspond to the probability of each value.
6. Make sure the bars do not touch, since the data is discrete.

Example 5:

Construct a probability distribution showing the number of heads that will appear when tossing a coin three times and draw the graph.

Solutions:

- **Step 1:** Identify the random variable

  Let X= number of heads in 3-coin tosses.

  Possible values of X are:

  $$0,1,2,3$$

- **Step 2:** List all possible outcomes

  There are $2^3=8$ outcomes when tossing a coin three times:

  TTT, TTH, THT, THH, HTT, HTH, HHT, HHH

- **Step 3:** Count the number of heads for each outcome

Outcome	**Number of Heads** (X)
TTT	0
TTH	1
THT	1
THH	2
HTT	1
HTH	2
HHT	2
HHH	3

- **Step 4:** Determine probabilities

X **(Number of Heads)**	**Number of Outcomes**	**Probability** P(X)
0	1	1/8 = 0.125
1	3	3/8 = 0.375
2	3	3/8 = 0.375
3	1	1/8 = 0.125

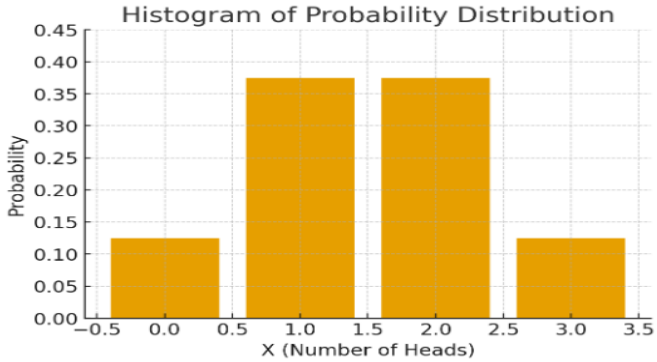
- **Step 5:** Probability Distribution Table

Heads (X)	0	1	2	3
P(X)	0.125	0.375	0.375	0.125

Check:

$$0.125+0.375+0.375+0.125=1\checkmark$$

Graph:



Example 6:

Construct a probability distribution table showing the probability of the number of even numbers in a single die roll, draw the graph.

Solutions:

- **Step 1:** Define the random variable

  Let Y= number of even numbers in one roll of a standard six-sided die.

  Possible values of Yare: 0,1

- **Step 2:** List possible outcomes

Die Outcome	**Is it Even?**	Y
1	No	0
2	Yes	1
3	No	0
4	Yes	1
5	No	0
6	Yes	1

- **Step 3:** Determine probabilities

Y **(Number of Even Numbers)**	**Number of Outcomes**	**Probability** P(Y)
0	3	3/6 = 0.5
1	3	3/6 = 0.5

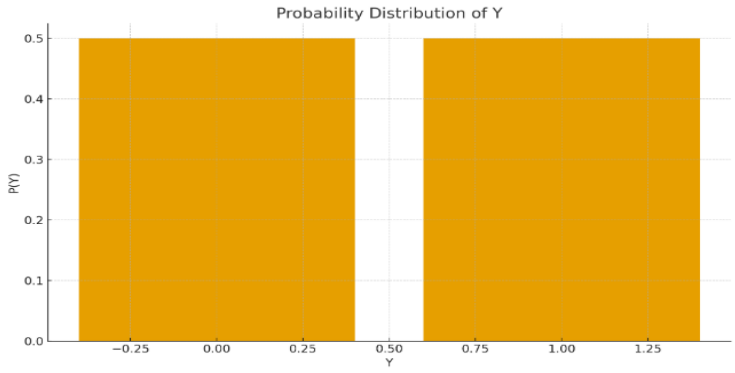
- **Step 4:** Probability Distribution Table

Y	P(Y)
0	0.5
1	0.5

Check:

$$0.5+0.5=1\checkmark$$

Graph:



Expected Value

The **expected value** or **mean** (denoted as *E(X)* or μ) of a random variable *X* is the average or mean value that ***X*** will take over many repetitions of an experiment. It represents the long-run average outcome.

Expected value of Discrete Random Variable

For a discrete random variable with values x1, x2,x3,..., xn and probabilities p1, p2,p3,..., pn:

$$E(X)=\sum X \cdot PX$$

where:

E(X) = expected value or mean

X = random variable

P(X) = probability of the random variable.

Example 7:

A fair six-sided die is rolled. Let X be the number that appears. Find the expected value of X.

Solutions:

A fair die has outcomes: 1, 2, 3, 4, 5, 6.

$E(X)=\sum X \cdot PX$

$EX=1 \cdot 16+2 \cdot 16+3 \cdot 16+4 \cdot 16+5 \cdot 16+6 \cdot 16$

$EX=1+2+3+4+5+66$

$EX=216$

$EX=3.5$

Answer: $E(X)=3.5$

Example 8:

A game costs ₱10 to play. A player draws a card from a box with 5 cards: 2 cards give ₱0, 2 cards give ₱20, and 1 card gives ₱50. Let X be the net gain (prize minus cost). Find the expected value of the game.

Solutions:

Possible net gains:

₱0 prize → X = -10 (probability 2/5)

₱20 prize → X = 10 (probability 2/5)

₱50 prize → X = 40 (probability 1/5)

$EX=-1025+10 \cdot 25+40 \cdot 15$

$EX=-4+4+8$

$EX=8$

Answer: Expected gain = ₱8.00

Example 9:

A survey shows the number of books learners read in a month as follows. Find the expected number of books a learner reads.

Number of books (X)	**0**	**1**	**2**	**3**	**4**
Probability P(X)	0.1	0.2	0.4	0.2	0.1

Solutions:

$E(X)=\sum X \cdot PX$

$EX=0 \cdot 0.1+1 \cdot 0.2+2 \cdot 0.4+3 \cdot 0.2+4 \cdot 0.1$

$EX=0+0.2+0.8+0.6+0.4$

$E(X)=2$

Answer: Expected number of books = 2

Variance and Standard Deviation of Discrete Random Variable

Variance is a measure of how much the values of a random variable deviate from the mean (expected value). It gives an idea of the spread or dispersion of the data. For a discrete random variable X with expected value E(X), the variance is calculated as:

$$Var(X)=\sum(X-E(X))2 \cdot P(X)$$

where:

X = possible value of the random variable

E(X)= expected value (mean) of X

PX= probability of X

A higher variance indicates that the values are more spread out from the mean, while a lower variance indicates that they are closer to the mean.

Standard deviation is the square root of the variance. It is expressed in the same units as the random variable, making it easier to interpret.

$$SD(X)=Var(X)$$

Like variance, a higher standard deviation means the data is more spread out, while a lower standard deviation means the data is more concentrated around the mean.

In short:

Variance measures spread in squared units.

Standard deviation measures spread in original units, making it easier to understand and compare.

Example 10:

Determine the variance and standard deviation of Example 7, 8, and 9.

Solutions:

- *Variance and Standard Deviation of Example 7*

Problem Recap:

Let X = number appearing on a fair six-sided die.

- **Step 1.** Expected Value (Mean)

$$E(X)=3.5$$

- **Step 2.** Variance Formula

$$Var(X)=\sum(X-E(X))2\cdot P(X)$$

- **Step 3.** Calculate Variance

X	P(X)	X - E(X)	$(X - E(X))^2$	$(X - E(X))^2 \cdot P(X)$
1	1/6	-2.5	6.25	$6.25 \cdot 1/6 \approx 1.0417$
2	1/6	-1.5	2.25	$2.25 \cdot 1/6 \approx 0.3750$
3	1/6	-0.5	0.25	$0.25 \cdot 1/6 \approx 0.0417$
4	1/6	0.5	0.25	$0.25 \cdot 1/6 \approx 0.0417$
5	1/6	1.5	2.25	$2.25 \cdot 1/6 \approx 0.3750$
6	1/6	2.5	6.25	$6.25 \cdot 1/6 \approx 1.0417$

$VarX=1.0417+0.3750+0.0417+0.0417+0.3750+1.0417$
$VarX=2.9168$
$VarX\approx2.92$

- **Step 4.** Standard Deviation

$SDX=VarX$
$SDX=2.9168$
$SDX\approx1.71$

Answer: Variance ≈ 2.92
SD ≈ 1.71

- *Variance and Standard Deviation of Example 8*

Problem Recap:

Net gain X = -10, 10, 40 with probabilities 2/5, 2/5, 1/5

- **Step 1.** Expected Value

$$E(X)=(-10)\cdot25+10\cdot25+40\cdot15=8$$

- **Step 2.** Variance Formula

$$Var(X)=\sum(X-E(X))2\cdot P(X)$$

- **Step 3.** Calculate Variance

X	P(X)	X - E(X)	$(X - E(X))^2$	$(X - E(X))^2 \cdot P(X)$
-10	2/5	-18	324	$324 \cdot 2/5 = 129.6$
10	2/5	2	4	$4 \cdot 2/5 = 1.6$
40	1/5	32	1024	$1024 \cdot 1/5 = 204.8$

$VarX=129.6+1.6+204.8$
$VarX=336$

- **Step 4.** Standard Deviation

$SDX=336$
$SDX\approx18.33$
Answer: Variance = 336
SD ≈ 18.33

- *Variance and Standard Deviation of Example 9*
- **Step 1.** Expected Value

Number of books (X)	0	1	2	3	4
Probability P(X)	0.1	0.2	0.4	0.2	0.1

$EX=0\cdot0.1+1\cdot0.2+2\cdot0.4+3\cdot0.2+4\cdot0.1$
$EX=2$

- **Step 2.** Variance Formula

$$Var(X)=\sum(X-E(X))2\cdot P(X)$$

- **Step 3.** Calculate Variance

X	P(X)	X - E(X)	$(X - E(X))^2$	$(X - E(X))^2 \cdot P(X)$
0	0.1	-2	4	4 0.1 = 0.4
1	0.2	-1	1	1 0.2 = 0.2
2	0.4	0	0	0 · 0.4 = 0
3	0.2	1	1	1 0.2 = 0.2
4	0.1	2	4	4 0.1 = 0.4

$VarX=0.4+0.2+0+0.2+0.4$
$VarX=1.2$

- **Step 4.** Standard Deviation

$SDX=1.2$
$SDX\approx1.095$
$SDX\approx1.10$
Answer: Variance = 1.2
SD ≈ 1.10

Data Analysis Using Expected Value, Variance and Standard Deviation

Example 11:
A player plays a simple board game where they can win or lose money based on the roll of a spinner. The outcomes are:

X **(Net Gain in ₱)**	-20	0	10	30
P(X)	0.2	0.3	0.4	0.1

Find the expected value, variance, and standard deviation of the net gain. Interpret the results.

Solutions:

- **Step 1.** Expected Value (Mean)

$EX=\sum X\cdot PX$
$EX=-200.2+00.3+100.4+300.1$
$EX=-4+0+4+3$
$EX=3$
Interpretation
On average, a player can expect to win ₱3.00 per game.

- **Step 2.** Variance

$$\text{Var}(X)=\sum(X-E(X))2\cdot P(X)$$

X	P(X)	X - E(X)	$(X - E(X))^2$	$(X - E(X))^2 \cdot P(X)$
-20	0.2	-23	529	$529 \cdot 0.2 = 105.8$
0	0.3	-3	9	$9 \cdot 0.3 = 2.7$
10	0.4	7	49	$49 \cdot 0.4 = 19.6$
30	0.1	27	729	$729 \cdot 0.1 = 72.9$

$$\text{Var}(X)=105.8+2.7+19.6+72.9=201$$

- **Step 3.** Standard Deviation

$\text{SDX}=\text{VarX}$
$\text{SDX}=201$
$\text{SDX}\approx 14.18$

- **Step 4.** Interpretation
  - *Expected value:* On average, the player wins ₱3.00 per game.
  - *Variance:* The variance of 201 indicates that there is a high spread in possible outcomes, meaning results can vary widely.
  - *Standard deviation:* The standard deviation of ≈₱14.18 implies that most of the game outcomes typically deviate around ₱14.00 from the mean, so the game is quite risky.

  It can be concluded that, while the player expects a small profit on average, the large standard deviation indicates high variability, meaning they could either lose significantly or win much more than the average.

Example 12:
A community-based cooperative plans to invest surplus funds into one of two local startup projects: an **Eco-Tourism Transport Service (Venture Alpha)** or a **Community Cold-Storage Facility (Venture Beta)**. Let's use our complete toolkit to complete a comparative risk evaluation.

Solutions:

- **Step 1: Identify the Distribution Profiles**

Venture Alpha Table Analysis:

Return X_A	**20**	**50**	**100**
Probability P_{X_A}	0.3	0.5	0.2

Venture Beta Table Analysis:

Return X_B	**10**	**60**	**90**
Probability P_{X_B}	0.1	0.7	0.2

Step 2: Calculate Expected Values (Means)

- **Venture Alpha:** $E_{X_A}=200.3+500.5+1000.2= 6 + 25 + 20 =$₱51.0 thousand
- **Venture Beta:** $E_{X_B}=100.1+600.7+900.2= 1 + 42 + 18 =$₱61.0 thousand

Step 3: Calculate Variances and Standard Deviations

Venture Alpha Spread Sheet $E_{X_A}=51$**:**

X_A	$P(X_A)$	$X_A - E(X_A)$	$(X_A - E(X_A))^2$	$(X_A - E(X_A))^2 \cdot P(X_A)$
20	0.3	-31	961	$961 \cdot 0.3 = 288.3$
50	0.5	-1	1	$1 \cdot 0.5 = 0.5$
100	0.2	49	2401	$2401 \cdot 0.2 = 480.2$

$\text{Var}(X_A)=288.3 + 0.5 + 480.2 = 769.0$

$\text{SD}_{X_A}=\text{Var}_{X_A}=769.027.73$ thousand

Venture Beta Spread Sheet $E\mathbf{XB}=61$**:**

$\mathbf{XB}$	$P(\mathbf{XB})$	$\mathbf{XB} - E(\mathbf{XB})$	$(\mathbf{XB} - E(\mathbf{XB}))^2$	$(\mathbf{XB} - E(\mathbf{XB}))^2 \cdot P(\mathbf{XB})$
10	0.1	-51	2601	$2601 \cdot 0.1 = 260.1$
60	0.7	-1	1	$1 \cdot 0.7 = 0.7$
90	0.2	29	841	$841 \cdot 0.2 = 168.2$

$\text{Var}(\mathbf{XB}) = 260.1 + 0.7 + 168.2 = 429.0$

$\text{SD}\mathbf{XB} = \text{Var}\mathbf{XB} = 429.020.71$ thousand

Step 4: Strategic Recommendation

The cooperative should select **Venture Beta**. It delivers a higher long-run expected return ₱61.0K vs. ₱51.0K and carries a lower operational risk profile, as proven by its smaller standard deviation ₱20.71K vs. ₱27.73K. This makes Venture Beta appears to be both less risky and more profitable on average for cooperative management.

Remember:

Understanding random variables and how to measure their average and variability helps us make sense of uncertainty in everyday life. By applying these concepts, we can better analyze situations, make informed decisions, and even predict outcomes in games, experiments, and real-world events.

Other Resources:
(Digital Modules and Textbook, Video Lesson)

Belecina, R. R., & E. S. (2016). *RBS statistics and probability* (1st ed., pp. 200–210). Rex Book Store, Inc.

Department of Education. (n.d.). *Senior high school statistics and probability modules*. Department of Education Philippines

I. Activity 1: **Practicing Learned Skills**

II. Materials:

pen
calculator
graphing paper
printed probability tables or scenario cards

III. Instructions:

Perform the indicated tasks. Write your answers on a sheet of paper.

A. Identify whether each random variable is Discrete (D) or Continuous (C).

1. Number of learners absent in a class
2. Time (in minutes) spent answering a test
3. Number of cars passing a toll booth in an hour
4. Weight of a newborn baby
5. Number of defective items in a box

B. Construct a probability distribution table for each probability experiment and draw the graph.

1. Number of heads in tossing a coin twice.
2. Getting odd numbers in rolling a die.
3. Number of tails in tossing three coins.
4. Getting a heart when drawing a card from a standard deck.
5. Sum of numbers on the two dice.

C. Find the *mean*, *variance*, and *standard deviation* of each problem, then interpret the results.

1. A learner buys snacks during recess. The number of snacks she buys per day (X) follows this distribution:

Number of snacks (X)	0	1	2
P(X)	0.2	0.5	0.3

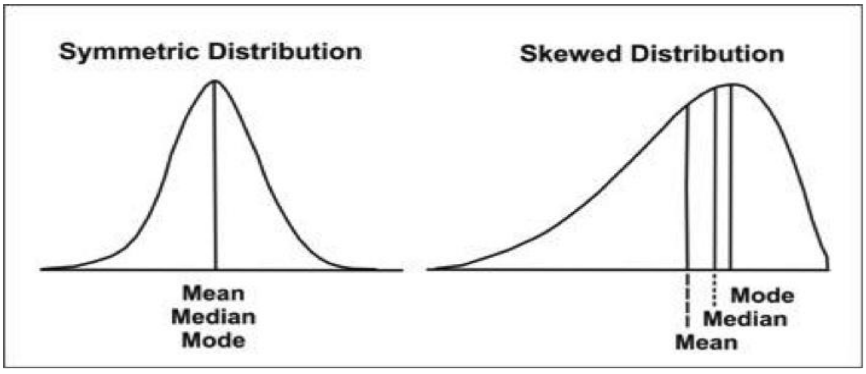
2. A tricycle driver observes the number of passengers picked up in one trip.

Passengers (X)	1	3	5
P(X)	0.4	0.4	0.2

	3. A customer service agent records the number of calls received in one hour. **Calls** (X) | 0 | 1 | 2 | 3 P(X) | 0.1 | 0.3 | 0.4 | 0.2 4. The number of attempts a learner needs to pass an online quiz is recorded. **Attempts** (X) | 2 | 4 | 6 P(X) | 0.5 | 0.3 | 0.2 5. A barangay health worker records the number of family members who received a flu vaccine in a day. The probability distribution is: **Family Members Vaccinated** (X) | 0 | 1 | 2 | 3 P(X) | 0.1 | 0.4 | 0.3 | 0.2
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. How did identifying the type of random variable help you answer the activity? 2. What pattern did you notice when constructing probability distributions? 3. What does the mean tell you about the given situations? 4. How does standard deviation help explain variability in the data?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper calculator
III. Instructions:	*Do the indicated task. Write your answer on a separate sheet of paper.* 1. A coin is tossed twice. Let X= number of heads. • Construct the probability distribution of X. • Find the expected value, variance, and standard deviation of X. 2. A fair six-sided die is rolled once. Let Y= 1 if an even number occurs, 0 if an odd number occurs. • Construct the probability distribution of Y. • Compute the expected value, variance, and standard deviation of Y. 3. A player draws a card from a standard deck of 52 cards. Let X= 1 if the card is a face card (Jack, Queen, King), 0 otherwise. • Construct the probability distribution of X. • Calculate the expected value, variance, and standard deviation. 4. A learner answers a multiple-choice question with 4 options. Let X= 2 points if correct, 0 if incorrect. The probability of answering correctly is 0.25. • Construct the probability distribution of X. • Find the expected value, variance, and standard deviation of the points earned. 5. A small lottery game offers the following prizes: ₱1000 with probability 0.05 ₱500 with probability 0.10 ₱100 with probability 0.20 ₱0 with probability 0.65 Let X= winnings per ticket. • Construct the probability distribution of X. • Determine the expected value, variance, and standard deviation. • Interpret the results in terms of average winnings and risk.
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. What steps did you follow in solving problems involving random variables? 2. How did identifying the type of variable help you solve the problem correctly? 3. Why is it important to interpret the mean and standard deviation in context? 4. What difficulties did you encounter while solving the activities, and how did you address them?

I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials	pen calculator graphing paper
III. Instructions:	*For each scenario:* 1. *Construct the probability distribution table for each scenario.* 2. *Compute Expected Value (E(X)), Variance (Var(X)), and Standard Deviation (SD(X)).* 3. *Interpret your results in context (e.g., risk, variability, or average outcome).* **For Remediation** 1. A coin is tossed once. Let X=1if heads, 0 if tails. X | 0 | 1 P(X) | 0.5 | 0.5 2. Roll a die. Let Y=1if number > 4, 0 otherwise. Y | 0 | 1 P(Y) | 4/6 | 2/6 3. A team can score 0, 1, or 2 goals in a game with probabilities 0.2, 0.5, 0.3 respectively. X | 0 | 1 | 2 P(X) | 0.2 | 0.5 | 0.3 4. A true/false question: X = 1 if correct, 0 if incorrect. Probability correct = 0.6. X | 0 | 1 P(X) | 0.4 | 0.6 5. A shop expects 0, 1, or 2 customers in an hour with probabilities 0.1, 0.6, 0.3. X | 0 | 1 | 2 P(X) | 0.1 | 0.6 | 0.3 **For Enhancement** 1. Roll two dice. Let X= sum of the dice. Construct probability distribution. 2. A small store expects daily sales of ₱0, ₱50, ₱100, ₱150 with probabilities 0.1, 0.3, 0.4, 0.2. 3. A spinner gives net gains: ₱-10 (0.2), ₱0 (0.3), ₱20 (0.4), ₱50 (0.1). 4. A quiz has 3 points max. Probability of scoring 0, 1, 2, 3 points: 0.1, 0.2, 0.4, 0.3. 5. A factory produces 0, 1, 2, 3 defective items per batch with probabilities 0.05, 0.25, 0.45, 0.25.
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. What strategy helped you solve the activity? 2. How did this activity improve your understanding of discrete random variables? 3. Why is it important to know the expected value and standard deviation of a random variable?

Lesson 10.2. The Normal Distribution	
Objectives:	At the end of the lesson, you are expected to: 1. describe and illustrate the shape, properties, and characteristics of a normal distribution curve. 2. apply the z-score formula to convert a normal random variable to a standard normal variable and vice versa. 3. determine probabilities and percentiles using the standard normal distribution table (z-table) or appropriate technologies (e.g., calculator, software). 4. solve and interpret real-life problems involving normal distribution.
Key Ideas and Examples:	**The Normal Distribution** The **normal distribution**, also called the **Gaussian distribution**, is one of the most important probability distributions in mathematics and statistics. It was formally developed in the 18th–19th century by mathematicians **Abraham de Moivre** and later **Carl Friedrich Gauss** while studying natural errors in measurements. It is a continuous probability distribution that is symmetric about the mean (μ). It is known for its **bell-shaped, symmetric curve**, where: • Most values cluster around the **mean** (average), • The probabilities decrease evenly as values move away from the mean, • The **mean, median, and mode are equal**. The normal distribution appears naturally in many real-life situations such as: • Heights and weights of people • Test scores • Measurement errors • Natural and social phenomena  Because many random processes tend to form this pattern (especially with large data sets), the normal distribution became a foundation of probability theory, statistics, and data analysis, and it is widely used in education, science, economics, and quality control. Consequently, the normal distribution helps us describe, analyze, and predict patterns in data that exhibit natural variation. **Characteristics of a Normal Distribution** A normal random variable has the following key characteristics: **1. Bell-shaped and Symmetric** • The graph looks like a bell. • The left side is a mirror image of the right side. This implies that the values are equally likely on both sides of the average. **2. Mean = Median = Mode** • The mean (average), median, and mode are all at the center of the distribution it means that the most common value and the average are the same.

3. Single Peak (Unimodal)

- The curve has one highest point. This means that there is one most frequent or typical value.

4. Asymptotic to the Horizontal Axis

- The curve never touches the x-axis**.**
- It extends infinitely in both directions which means that the extreme values are possible but very unlikely.

5. Defined by Two Parameters

A normal random variable is described using:

- Mean () – the center of the distribution
- Standard Deviation () – the spread or variability

Area Under the Normal Curve

1. The area under the curve represents the probability that a random variable (X) falls within a certain range.
2. The total area under the normal curve is equal to one (1).
3. The mean divides the normal distribution curve into two equal parts.
4. The area between two points is probability that X falls in that interval.

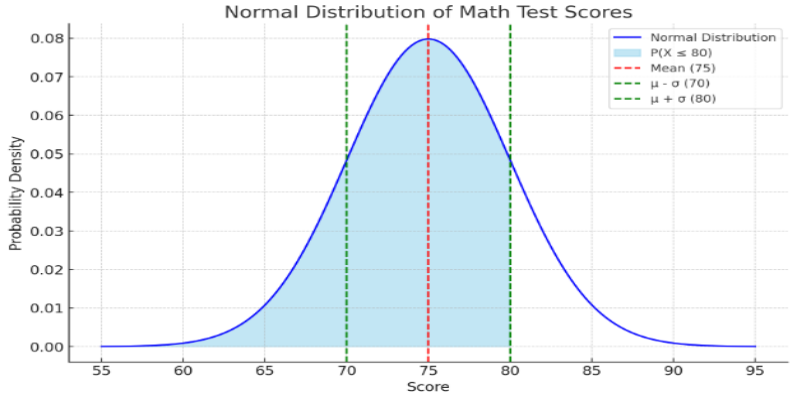
Steps to Sketch the Area Under the Normal Curve

1. **Draw the normal curve**:
   - Draw a smooth, bell-shaped curve symmetric about the mean ().
   - Mark the mean μ on the horizontal axis.

2. **Label key points**:
   - Mark $\mu \pm \sigma$ (one standard deviation away from the mean).

3. **Shade the area of interest**:
   - Identify the range for which you want the probability.
   - Shade the region under the curve corresponding to this range.

4. **Interpret the shaded area**:
   - The shaded area represents the probability of the random variable falling within that range.



Example 1.

Suppose the math test scores of a large group of learners are normally distributed.

- Mean score (μ) = 75
- Standard deviation (σ) = 5

Draw the normal distribution showing that the learners' score lower than 80.

The illustration above is the normal distribution curve for the math test scores:

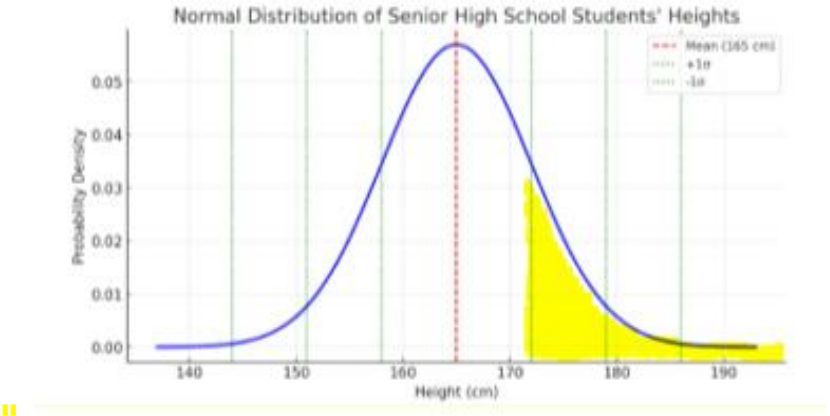
- The mean (75) is marked with a red dashed line at the center.
- One standard deviation from the mean (70 and 80) is marked with green dashed lines.
- The shaded area represents P(X≤80), the probability of a learner scoring 80 or less.

Example 2.

Let the heights of senior high school learners be normally distributed.

- Mean height = 165 cm
- Standard deviation = 7 cm

Draw the normal distribution showing that a learner's height is greater than 170 cm.

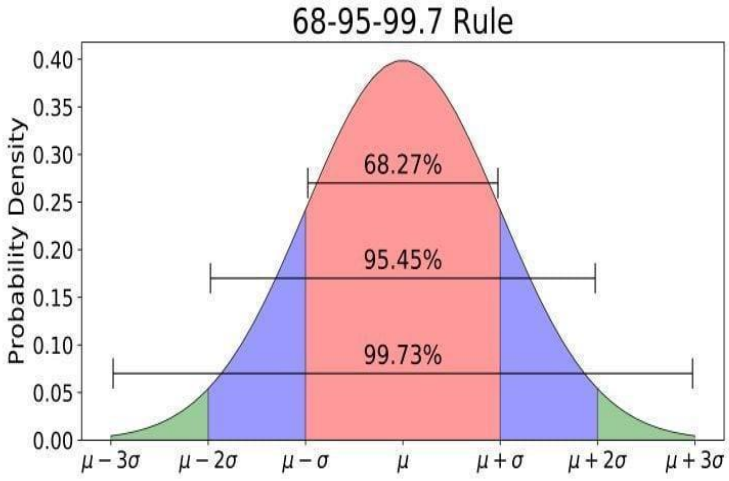


The illustration above is the normal distribution curve for the heights of senior high school learners:

- Mean (μ): 165 cm (red dashed line at the peak)
- Standard deviations (σ): 7 cm, shown as green dotted lines at $\pm 1\sigma, \pm 2\sigma, \pm 3\sigma$
- The curve is symmetric, with most learners' heights concentrated near the mean.
- The shaded area represents $P(X \geq 170)$, the probability that a learner's height is 170 cm or taller.

The Empirical Rule (68–95–99.7 Rule)

This rule helps describe how values are spread in a normal random variable.



Using the test score in Example 1

- About **68%** of learners scored between **70 and 80** $(\mu \pm 1\sigma)$
- About **95%** scored between **65 and 85** $(\mu \pm 2\sigma)$
- About **99.7%** scored between **60 and 90** $(\mu \pm 3\sigma)$

The Standard Normal Distribution

A **normal random variable** X has:

- Mean:
- Standard deviation:

Its probability distribution is called a normal distribution.

A **standard normal distribution denoted by** Z is a special case of the normal distribution with:

- Mean: $\mu=0$
- Standard deviation: $\sigma=1$

Why convert?

Converting X to Z allows us to use standard normal tables (Z-tables) to find probabilities easily.

Formula for Conversion

- **Converting** X **to** Z

$$Z=X-\mu$$

  - X= value of the normal random variable
  - = mean
  - = standard deviation
  - Z= corresponding standard normal value

Interpretation:

Z tells how many **standard deviations** X **is away from the mean**.

- **Converting** Z **to** X

$$X=Z\cdot\sigma+\mu$$

  - Z= standard normal value
  - = mean
  - = standard deviation
  - X= value of the original normal variable

Example 3. Converting X to Z

The heights of learners are normally distributed with $\mu=165$ cm,$\sigma=7$ cm.

Find the standard normal variable Z corresponding to a learner with **height *172 cm***.

Solution:

$Z=X-\mu$

$Z=172-1657$

$Z=77$

$Z=1$

Interpretation:

The height 172 cm is 1 standard deviation above the mean.

Example 4. Converting Z to X

Find the height of a learner corresponding to Z=-1.5.

Solution:

$X=Z\cdot\sigma+\mu$

$X=-1.57+165$

$X=-10.5+165$

$X=154.5$ cm

Interpretation:

A learner with Z=-1.5 has a height 1.5 standard deviations below the mean, which is 154.5 cm.

The Z-Table

A Z-**table** (or standard normal table) shows the area (probability) under the standard normal distribution curve to the left of a given Z-value.

Recall:

- A Z-value tells how many standard deviations a value is from the mean.
- The standard normal distribution has:
  - Mean = 0
  - Standard deviation = 1
- The total area under the curve = 1.00 (100%)

The Z-table helps us answer probability questions once we convert a normal random variable X into a standard normal variable Z.

- **Structure of the** Z**-Table**

The Z-table is usually arranged as follows:

- **Left column**: the whole number and first decimal of the Z-value
- **Top row**: the second decimal of the Z-value
- **Body of the table**: the area to the left of that Z-value

- **How to Read the Z-Table**

Example 5.
Find the value corresponding to Z=1.23.
Step 1: Identify the Z-value

$$Z=1.23$$

Step 2: Locate the row

Row = 1.2

Step 3: Locate the column

Column = 0.03

Step 4: Read the value

Table value ≈ 0.8907

Interpretation:

$$P(Z\leq 1.23)=0.8907$$

This means 89.07% of the values lie **below** Z = 1.23.

- **Understanding Areas Using the *Z*-Table**

A. Probability to the Left
Directly read from the Z-table.
Example 6.

$$P(Z\leq 0.50)=0.6915$$

B. Probability to the Right
Since total area = 1:

$$P(Z>a)=1-P(Z\leq a)$$

Example 7.

$$P(Z>1.25)=1-0.8944=0.1056$$

C. Probability Between Two Z-Values

$$P(a<Z<b)=P(Z\leq b)-P(Z\leq a)$$

Example 8.
Find $P(0.50<Z<1.50)$
From the table:

- $P(Z\leq1.50)=0.9332$
- $PZ\leq0.50=0.6915$

$P=0.9332-0.6915=0.2417$

About 24.17% of the values lie between $Z = 0.50$ and $Z = 1.50$.

- **Z-Table with Negative Z-Values**

The standard normal curve is **symmetric** about 0.
Key idea:

$$P(Z\leq-a)=1-P(Z\leq a)$$

Example 9.
Find $P(Z\leq-1.20)$
From table:
$PZ\leq1.20=0.8849$
$PZ\leq-1.20=1-0.8849$
$PZ\leq-1.20=0.1151$

About 11.51% lie below $Z = -1.20$.

- **Word Problems**

Example 10.
The heights of learners are normally distributed with $\mu=165$" cm", $\sigma=7$" cm". What is the probability that a learner is taller than 172 cm?

Step 1: Convert to Z

$$Z=172-1657=1$$

Step 2: Use the Z-table

$$P(Z\leq1)=0.8413$$

Step 3: Probability taller

$$P(X>172)=1-0.8413=0.1587\approx16\%$$

Example 11.
Test scores are normally distributed with $\mu=75$, $\sigma=5$.
Find the probability that a learner scores **above** 80.

Step 1: Convert X to Z

$$Z=80-755=1$$

Step 2: Use the Z-table

$$P(Z\leq1.00)=0.8413$$

Step 3: Find probability above

$$PX>80=1-0.8413$$
$$PX>80=0.1587$$

About 15.87% of learners scored above 80.

Example 12.
Suppose the math test scores of a large group of senior high school learners are normally distributed.
Mean score () = 75
Standard deviation () = 5

The school wants to know the proportion of learners whose scores are neither too low nor too high, specifically those who scored between 72.5 and 81 points.

Solutions:
Step 1: Convert the scores to Z-values
For 72.5:

$$Z=72.5-755=-0.5$$

For 81:

$$Z=81-755=1.2$$

Step 2: State the probability in terms of Z

$$P(72.5\leq X\leq 81)=P(-0.5\leq Z\leq 1.2)$$

Step 3: Use the Z-table
From the Z-table:

$$P(Z\leq 1.2)=0.8849P(Z\leq -0.5)=0.3085$$

Step 4: Compute the probability

$$P(-0.5\leq Z\leq 1.2)=0.8849-0.3085=0.5764$$

Interpretation
About 57.64% of the learners scored between 72.5 and 81 on the math test. This means more than half of the learners fall within this "average to above-average" score range.

Example 13.
A local SHS entrepreneurial cooperative produces banana chips. The net weight of the packs is normally distributed with a mean () of 150 grams and a standard deviation () of 8 grams. Distributors reject packages that are under-filled or over-filled, demanding packs only between 142 grams and 162 grams.

Tasks
A. Compute the percentage of production accepted by distributors.
B. Analyze the financial impact on the cooperative if the standard deviation fluctuates to 12 grams due to poor packaging machinery maintenance.

Solution:
Task A
Step 1. Convert the salaries to Z-values
For 142 **grams**:
$Z=142-1508=-88=-1.0$
For 162 **grams**:
$Z=162-1508=128=1.5$
Step 2. Express the probability in terms of Z
$P(142\leq X\leq 162)=P(-1.0\leq Z\leq 1.5)$
Step 3. Use the Z-table
From the Z-table:
$PZ\leq 1.5=0.9332$
$P(Z\leq -1.0)=0.1587$
Step 4. Compute the probability
$P-1.0\leq Z\leq 1.5=0.9332-0.1587$
$P-1.0\leq Z\leq 1.5=0.7745$

Interpretation
About **77.45%** of the banana chip production is accepted by the distributors. This means that under optimal machinery maintenance, approximately 22.55% of the produced packs are rejected as defective waste (either under-filled or over-filled).

	Task B Step 1. Convert the salaries to Z-values For 142 **grams**: Z=142-15012=-812≈-0.67 For 162 **grams**: Z=162-15012=1212=1.0 *Step 2. Express the probability in terms of* **Z** P(142≤X≤162)=P(-0.67≤Z≤1.0) *Step 3. Use the Z-table* From the Z-table: PZ≤1.0=0.8413 P(Z≤-0.67)=0.2514 Step 4. Compute the probability P-0.67≤Z≤1.0=0.8413-0.2514 P-0.67≤Z≤1.0=0.5899 Interpretation When machine maintenance is neglected, the product weight varies too much. This causes the standard deviation to expand from 8g to 12g, and the percentage of accepted products drops heavily from **77.45% down to 58.99%**. **Financial Risk Assessment:** This shift forces the cooperative's waste or rejection rate to spike from **22.55% to 41.01%**. Nearly half of the factory's output will now be turned away by distributors. In a student business setup, losing this much inventory leads directly to a drain on capital. The cooperative will not be able to recover basic raw material costs (bananas, oil, sugar, and packaging) and could face bankruptcy due to poor quality control. This shows why keeping variance as small as possible is critical to keeping a business alive. By completing these activities, you now understand how normal random variables help describe real-world data, how to use the standard normal distribution to calculate probabilities, and how to interpret results. Remember, these concepts are not just theoretical—they are powerful tools for analyzing patterns in education, science, business, and daily life.
Other Resources: (*Digital Modules and Textbook, Video Lesson)*	https://youtu.be/rzFX5NWojp0 https://youtu.be/2tuBREK_mgE https://youtu.be/CjF_yQ2N638

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen calculator graphing paper Z-table
III. Instructions:	*Answer the preliminary questions first to review the concepts needed for the activity. Then, complete the main task by solving each problem on a separate sheet of paper. Show all formulas, computations, and final answers, and provide solutions in an organized and accurate manner.* **Part A. Preliminary Questions:** Before answering the main tasks, respond to these preliminary questions: 1. What formula is used to convert a raw score X into a Z-score? 2. If X=μ, what Z-score will this correspond to? 3. What does the Z-score tell us about the position of X relative to the mean? 4. When solving for probabilities greater than a score (e.g., PX>x), do we directly use the Z-table value or subtract it from 1? Why? 5. What does the phrase "lowest 25% of learners" mean in terms of area under the normal curve? 6. From the Z-table, what approximate Z-score corresponds to the 25th percentile (0.2500 area)? 7. How do you convert a Z-score back into a raw score X?

	Part B. 1. On graph paper, sketch a bell curve for test scores with mean = 500 and standard deviation = 100. 2. Mark the intervals: 400–600, 300–700, and 200–800. 3. Shade each region and label the approximate % of data it covers. **Part C**. 1. Compute z for these cases: a. $X = 70,\ \mu = 60,\ \sigma = 5$b. $X = 45,\ \mu = 50,\ \sigma = 10$ 2. Convert back to raw scores X given: a. $z = 2,\ \mu = 100,\ \sigma = 15$b. $z = -1.2,\ \mu = 80,\ \sigma = 8$ **Part D.**. Sketch a quick normal distribution curve for each scenario and shade the target region based on visual logic: 1. For the expression $PZ>1.5$, should your shaded region cover more than half or less than half of the total bell curve area? Why? 2. If $PZ>a=0.15$, will the boundary value of 'a' sit on the positive (right) side or negative (left) side of the distribution peak? Explain using curve symmetry. **Part E.** Read the given scenarios and complete the task given: • *Scenario 1* IQ scores are normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 15$. *Tasks:* Find the following probabilities: a. $PX > 115$ b. $PX < 85$ c. $P(85 < X < 115)$ • *Scenario 2* SAT scores are normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 100$. *Tasks:* Answer the following questions: a. What percentage of learners score above 650? b. What score marks the lowest 25% of learners? • *Scenario 3* An SHS student takes two technical assessment tests for employment certification. On Test 1 $\mu=80$, $\sigma=6$, they score 89. On Test 2 $\mu=500$, $\sigma=50$, they score 565. The employer only hires applicants who rank in the top 5% of either test pool. *Tasks:* Calculate the Z-scores, use the Z-table, and defend which test results give the student the best chance at landing the job.
V. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* 1. Why is the normal distribution useful in analyzing test scores? 2. What does it mean if a student has a high positive z-score? 3. Why do we use the z-table in statistics? 4. How does standard deviation affect the shape of the curve?

I. Activity 2:	Assessing Learning Outcomes
II. Materials:	pen calculator Z-table
III. Instructions:	**Part I** ***Instructions:*** • Without performing calculations, draw a clean standard normal distribution curve. • Label the exact positions of the mean, median, and mode along the horizontal axis. • Write a short 2-sentence explanation describing why the curve is considered *asymptotic* to the axis and what that implies about the possibility of encountering extreme outlier data points in real-life measurements. **Part II** ***Instructions:*** Read each scenario carefully and perform the task as directed**.** 1. A test has $\mu = 50$ and $\sigma = 8$. If a learner scores 66, what is the z-score? 2. If $z = -1.25$, $\mu = 100$, $\sigma = 12$, what is the raw score? 3. The scores on a standardized test are normally distributed with a mean of 500 and a standard deviation of 90. Determine: a. $P(X < 560)$ b. $P(X > 440)$ 4. The weights of bags of rice are normally distributed with $\mu = 50$ kg and $\sigma = 2$ kg. a. What percentage of the bags weigh more than 53 kg? b. A retailer only accepts bags in the middle 80% of weights. What is the acceptable weight range?
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. What steps did you follow to solve probability problems involving normal distribution? 2. How did you interpret the probability you obtained? 3. What challenges did you encounter while solving the problems? 4. How can you check if your answer is reasonable?

I. Activity 3:	Extending and Reinforcing Learning
II. Materials	pen calculator Z-table graphing paper
III. Instructions:	**For Remediation** *Before starting your calculations, write down the formula $z = X - \mu$. Identify and list your given values first: What is your raw score (X)? What is your mean (μ)? What is your standard deviation (σ)?* 1. Given a normal distribution with a mean of 50 and a standard deviation of 5, find the Z-score that corresponds to a raw score of 60. 2. Find the raw X when the mean is 100, the standard deviation is 15, and the Z-score is -2.0. 3. IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the probability that a randomly selected individual has an IQ score lesser than 85. 4. The number of hours high school students spend online per week is normally distributed with a mean of 200 hours and a standard deviation of 20 hours. a. What percentage of students spend more than 220 hours online per week? b. Determine the cutoff score for the bottom 5% of online users.

	For Enhancement *Solve the following advanced analytical problems. Show your complete step-by-step solutions and justifications on a separate sheet of paper.* 1. Examine the two normal distribution curves below: Curve A (blue) and Curve B (red). Both curves share the exact same mean , but they have different standard deviations . a. Draw two normal curves on the same graph: • Curve A: $\mu = 170$ cm, $\sigma = 5$ cm (learners' heights in School A). • Curve B: $\mu = 170$ cm, $\sigma = 10$ cm (learners' heights in School B). b. Label both means. b. Compare: Which curve is narrower? Which is wider? What does this tell you about variation in heights? b. If a student standing 185cm tall transfers into both schools, in which school would their height be considered more unusual or extreme? Explain using the concept of standard deviation. 2. Compare the performance of the two learners relative to their respective groups. Who performed better, and why? Consider the mean, standard deviation, and each learner's score in your explanation *Alice: Test A → $\mu = 70$, $\sigma = 8$, score = 82.* *Ben: Test B → $\mu = 500$, $\sigma = 100$, score = 630.* • Calculate the standardized Z-scores for both Alice and Ben, determine who holds the higher relative rank within their specific testing pool, and explain why comparing raw scores directly would lead to an incorrect decision. 3. Analyze the situation below and answer the questions that follow. The SAT scores are normally distributed with a mean of 500 and a standard deviation of 90. a. The top 5% of learners receive a scholarship. What is the minimum SAT score needed? b. What exact percentage of learners score between 450 and 600?
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. What steps did you follow to solve each normal distribution problem? 2. How did you decide which formula or table to use for each scenario? 3. What difficulties did you encounter, and how did you address them? 4. Why is it important to use Z-scores when comparing results from different tests or groups?

Lesson 10.3: Sampling	
Objectives:	At the end of the lesson, you are expected to: 1. define random sampling and its types. 2. recommend appropriate sample size for a given investigation. 3. differentiate between a parameter and a statistic. 4. compare probability and non-probability sampling techniques based on their characteristics and applications.
Key Ideas and Examples:	**Sampling** **Sampling** is the process of selecting a subset (called a sample) from a larger group (called a population) to gather information and draw conclusions without examining the entire population. This activity introduces key sampling concepts such as random sampling, sample size, parameters vs. statistics, and the comparison between probability and non-probability sampling methods. **Determining Appropriate Sample Size** In sampling, the first thing that you must consider is the sample size. In any research, you usually cannot study the entire population because it may be too large, expensive, or time-consuming. Instead, researchers select a sample that represents the population.

You must think:

"How many people or items should you include in your study to obtain useful results? Having enough participants to observe trends or patterns should always be considered in an investigation to avoid wasting time or resources."

Factors Affecting Sample Size

Even without statistical formulas, sample size depends on:

• **Purpose of the study**	Are you just exploring opinions, or do you want to compare groups?
• **Population size**	How many people or items exist in total?
• **Resources available**	Time, money, or workforce may limit how many participants you can include
• **Type of research**	Surveys, experiments, or qualitative interviews each have different practical requirements.

Simple Ways to Decide Sample Size

A. Rule-of-Thumb Estimates

These rules-of-thumb are based on experience, simulation studies, and conventional practices in research, rather than a single published formula.

The purpose is to provide quick, practical estimates for researchers and educators, especially for teaching or initial planning, before using formal formulas.

Type of Research	Suggested Sample Size
Survey (large population)	200–400 respondents for a general idea
Small group survey	At least 20–50 respondents
Experimental research (2 groups)	15–30 participants per group
Qualitative research (interviews)	10–20 participants, or until no new information emerges (saturation)

B. Using a Simplified Formula

If the population is known and small, you can use a simple "proportion-based" formula:

$$\text{Sample size} = \text{Population} 2 \text{ or more}$$

Example:

A class has 60 learners. A teacher wants to survey about class preferences. A simple choice is 30 learners (half of the class). If more precision is needed, you can increase the sample to two-thirds 23 of the population.

C. Using Slovin's Formula

For a more scientific estimate, researchers may use Slovin's Formula.

$$n = N1 + Ne2$$

Where:

- **n** = sample size
- **N** = population size
- **e** = margin of error

Example:

A school has 1,000 learners. The researcher wants a 5% margin of error.

Solution:

$n = N1 + Ne2$
$n = 10001 + 10000.052$
$n = 10001 + 2.5$
$n = 10003.5$
$n = 285.71$
$n \approx 286$

Therefore, the recommended sample size is **286 learners**.

D. Using Technology

You can use online sample size calculators or spreadsheet tools:

Online calculators	Just enter population size and desired margin of error • Raosoft Sample Size Calculator • SurveyMonkey Sample Size Calculator
Spreadsheet method	Create a column of participants, then randomly select a number based on available resources or rules of thumb.

The Random Sampling

Random sampling means that every individual in the population has an equal chance of being selected. This method is important because it:

a. Reduces bias: No one is unfairly favored or left out.

b. Increases representativeness: The sample better reflects the diversity of the whole population.

c. Improves reliability of results: Conclusions drawn from a random sample are more likely to be valid for the whole population.

Example:

If you have 500 learners and want to select 50 randomly, you might use a computer to generate 50 random numbers representing learner IDs.

Types of Sampling

I. Probability Sampling

Every individual has a known chance of being chosen. Probability sampling is generally preferred because it leads to more accurate and generalizable results, while non-probability sampling can lead to bias.

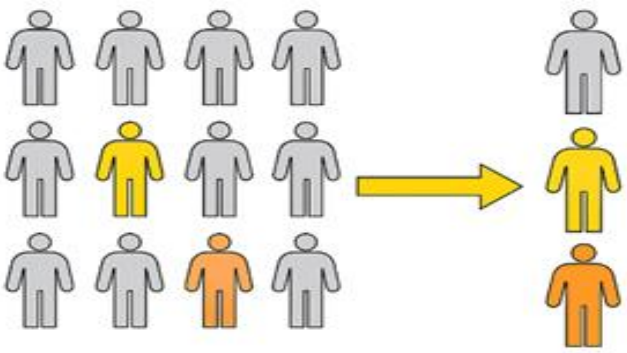
Examples include:

a. Simple Random Sampling:

Everyone is equally likely to be picked (e.g., randomly drawing names or using a random number generator).

For example, in a population of 200 Grade 11 learners, the researcher needs a sample of 40 learners. Using simple random sampling, every learner in the population has an equal chance of being chosen. The researcher assigns numbers from 1 to 200 to all learners and then uses a random method, such as drawing lots or using a random number generator, to select 40 unique numbers. The learners correspond to those numbers form the sample.

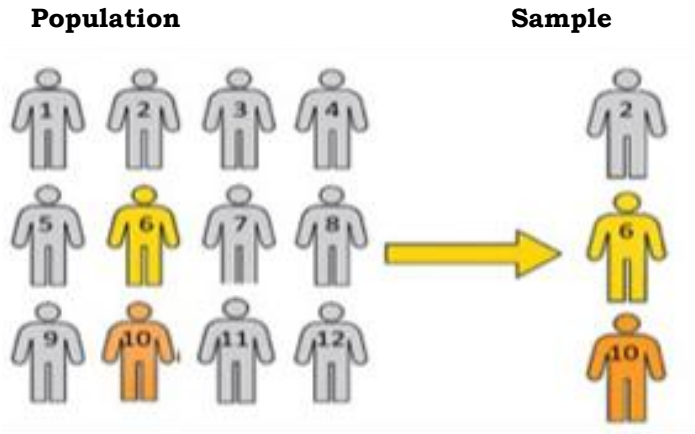
Population **Sample**



b. Systematic Sampling:

Selecting every kth individual from a list (e.g., every 4th learner). The starting point is chosen at random.

For example, in 200 Grade 12 learners, you will choose 50 learners as members of your sample. To select which learners will be part of the sample, divide the population size 200 by the sample size 50, which results in 4. That means every 4th learner among the 200 Grade 12 learners will be included in the sample. You may also start with any randomly selected number from 1 to 4; for instance, if the starting number is 2, then the sample will include learners numbered 2, 6, 10, and so on, until the desired 50 learners are chosen. (Note: Other starting points may also be used.)

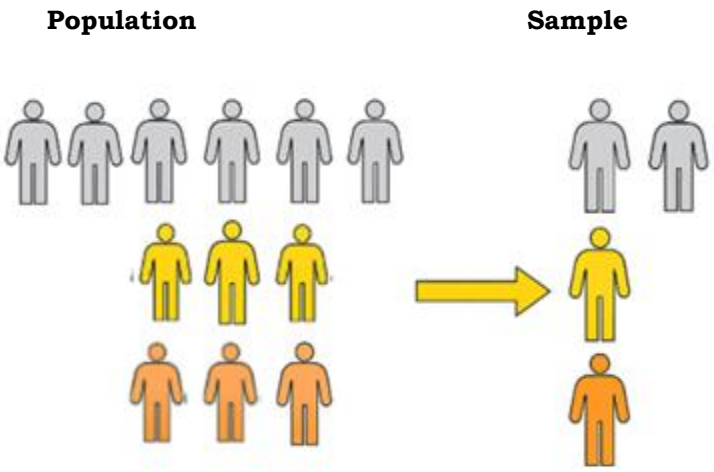


c. Stratified Sampling:

The population is divided into subgroups or strata (e.g., by grade level or strand). Samples are randomly selected from each stratum to ensure all groups are represented.

For example, in a population of 300 Grade 11 learners composed of three strands: 150 STEM learners, 90 HUMSS learners, and 60 GAS learners, the researcher needs a sample of 60 learners. To make sure each strand is fairly represented, the researcher uses stratified sampling. First, the proportion of each group is determined: STEM is 150/300 = 50%, HUMSS is 90/300 = 30%, and GAS is 60/300 = 20%. Next, these same proportions are applied to the sample size of 60. Thus, the researcher will select 30 STEM learners (50% of 60), 18 HUMSS learners (30% of 60), and 12 GAS learners (20% of 60).

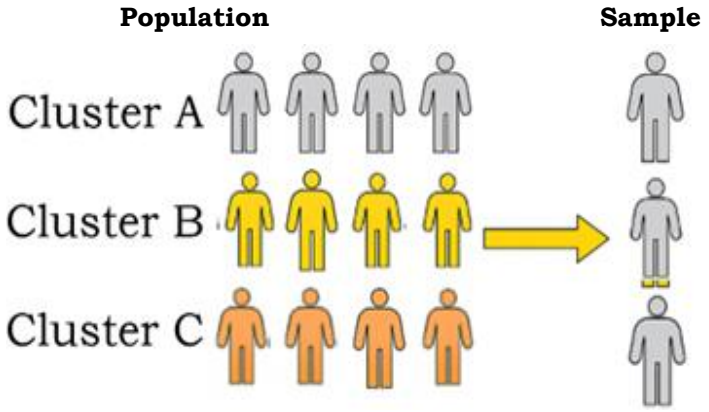
Finally, the learners are chosen until the required number of learners from each strand is then selected randomly.



d. Cluster Sampling:

The population is divided into clusters, usually based on location or existing groups (e.g., classes or barangays). A few clusters are randomly selected, and all individuals within those clusters are surveyed.

For example, in a population of 150 Grade 11 learners, the researcher needs a sample of 50 learners. The 150 learners are already divided into 3 sections with 50 learners each. Using cluster sampling, the researcher randomly selects 1 section (cluster) out of the 3. All 50 learners in the chosen section automatically become the sample.



Cluster A is randomly chosen as sample.

II. Non-probability Sampling

Selection is based on convenience or judgment, not chance.

Examples include:

a. Convenience Sampling

Choosing whoever is easiest to access (e.g., the first 50 learners who enter the cafeteria).

b. Purposive (Judgmental) Sampling

Choosing individuals who possess specific characteristics relevant to the study.
Example: Interviewing only science teachers when studying teaching strategies in science.

c. Quota Sampling

The population is divided into groups, and a predetermined number of participants is selected from each group, though the selection is non-random.
Example: Surveying 20 male and 20 female learners to reflect gender balance.

d. Snowball Sampling

This technique relies on existing participants to recruit additional participants, creating a referral chain.
Example: A learner respondent recommends friends, who in turn recommend others, to join the study.

e. Voluntary Response Sampling

In this type, individuals volunteer or decide by themselves to participate in the research.
Example: Learners who willingly respond to an online survey posted by the school.

	Parameter vs. Statistic These two terms describe similar measurements but at different levels: • **Parameter**: A value that describes a characteristic of the entire population (e.g., the average snack preference of all 500 learners). This is usually unknown because it's hard to collect data from everyone. • **Statistic**: A value that describes the same characteristic but based on the sample (e.g., the average snack preference of the 50 learners surveyed). Statistics are used to estimate parameters. The goal of sampling is to use statistics to make reasonable guesses about parameters.
Other Resources: *(Digital Modules and Textbook, Video Lesson)*	Statistics and Probability – Sampling Techniques (https://www.youtube.com/watch?v=5PsF5MsrCOo

I. Activity 1:	**Practicing Learned Skills**
II. Materials:	pen paper
III. Instructions:	**Option 1. Classroom Simulation** Part I. *Complete each sentence.* 1. A _____ is the entire group being studied 2. A _____ is a smaller part of the population 3. A _____ is a numerical characteristic of a population. 4. A _____ is a characteristic of a sample. 5. _____ means everyone has an equal chance of being chosen. Part II. **Show Us What You Got!** 1. Each group will act out one sampling method using classmates or name cards. 2. Decide if it is probability or non-probability sampling. 3. Explain why sample size and method matter. • Does your method give accurate results? • Is it quick and easy, or fair and representative? Part III. **Sample It Right!** 1. Each group will be provided with scenario cards describing different sampling situations a. A teacher wants to know the preferred mobile phone brand of all Grade 11 learners in your school. Every learner should have an equal chance of being selected. b. The librarian wants to survey learners about their reading habits. She lines up all the learners who enter the library in one week and selects every 5th learner to answer the survey. c. The learner council wants to survey 100 learners about their study habits. Since the school has three strands (STEM, HUMSS, GAS), they want to make sure each strand is fairly represented in the sample. d. A barangay health worker wants to survey families about waste management. Instead of selecting households from all over the barangay, she randomly chooses two whole streets and surveys every household in them. e. A learner researcher wants quick feedback about the cafeteria food. He surveys only the learners sitting at the nearest tables during lunchtime. f. A guidance counselor wants to understand the experiences of learners with anxiety. She interviews one learner, who then refers her to another learner with the same concern, and the process continues. g. A science teacher wants to study how top-performing learners prepare for exams. She chooses only the honor learners from each section to participate in the survey.

h. A researcher wants to survey 60 learners about their social media habits. To ensure balance, she decides to collect responses from exactly 30 Academic and 30 TechPro learners, regardless of how many are available.

2. The group will identify the population, choose an appropriate sampling method, and decide on sample size.
3. Your group will present the sampling strategies and justify your choices.

Option 2. Sampling Challenge

Part I. **Matching Race**

Work with a partner to match each term with its correct definition using the given cards. Complete the activity within 2 minutes and be prepared to check your answers with the class. The fastest pair with all correct matches earns points or praise.

Terms	**Definitions**
Parameter	The entire group being studied
Population	A smaller part of the population
Random Sampling	A characteristic of the population
Sample	A characteristic of the sample
Statistic	Everyone has an equal chance of being chosen

Part II. **Case Challenge**

Read each case scenario carefully, identify the sampling method being used, and discuss your answers with a partner or in a small group before sharing them with the class.

Case Scenarios:

1. A teacher wants to know the learning style of all. A learner quickly surveys classmates in the canteen.
2. Grade 11 learners.
3. A clinic wants to survey learners with asthma.
4. A researcher wants fair representation from all tracks (Academic, TechPro).
5. A barangay officer surveys whole blocks of houses.

Part III. **Sampling Showdown**

1. Look back at the five case scenarios from Part II.
2. Decide which ones are probability sampling and which are non-probability sampling.
3. Share your answers with your partner.
4. Be ready to explain your choices during the class discussion.

Option 3. Sampling Face-Off!

Part I. **Hot Seat Game**

Take turns sitting in the hot seat and answer the teacher's question within 10 seconds. Each correct answer earns a point for your group; if you answer incorrectly or run out of time, another learner will have the opportunity to answer.

Hot Seat Questions

1. What is a parameter?
2. What is a statistic?
3. What is random sampling?
4. What is the difference between population and sample?
5. Why is random sampling important?

	Part II. **Face-Off** 1. The class will be divided into two teams: • Team A - Argues for Probability Sampling • Team B - Argues for Non-probability Sampling 2. Each team will have 10 minutes to prepare arguments. 3. Debate Rules: • Opening Arguments: Each team speaks for 3 minutes. • Rebuttals: Each team has 2 minutes to respond to the other side. • Q&A: Teams may ask each other questions for 5 minutes. 4. Be respectful, listen carefully, and support your answers with reasons or examples.
IV. Reflection:	*Reflect on your experience and learning from the activity by answering the following questions.* **Option 1: Classroom Simulation** 1. How did your group decide which sampling method was being used in the activity? 2. Why do you think sample size and sampling method are important in getting results? 3. What makes a sampling method fair or unfair? **Option 2: Sampling Challenge** 1. How did you identify which sampling method was used in each case? 2. Which sampling method do you think gives the most reliable results? Why? 3. What might happen if the wrong sampling method is used? **Option 3: Sampling Face-Off!** 1. What is the difference between probability and non-probability sampling based on the activity? 2. Which sampling method do you think is better for school research? Why? 3. If you were doing a school survey, what sampling method would you use and why?

I. Activity 2:	**Assessing Learning Outcomes**
II. Materials:	pen and paper
III. Instructions:	Read each item carefully. Choose the best answer and write only the letter of your choice. Part I. Concepts in Sampling 1. Which best defines a parameter? A. Sample characteristic B. Population characteristic 2. Surveying the first 20 students you see is an example of: A. Random sampling B. Convenience sampling 3. Which method gives every learner an equal chance of being selected? A. Purposive sampling B. Simple random sampling 3. Selecting learners from different strands in proportion to their population is: A. Cluster sampling B. Stratified sampling 4. Why is non-probability sampling less reliable? A. It is more expensive B. It may introduce selection bias 5. The average score of a sample of 30 students is a: A. Statistic B. Parameter

	Part II. Sample Size and Sampling 1. If N = 500 and e = 0.05, what is the sample size using Slovin's Formula? A. 222 B. 50 3. Selecting every 10th name on a list is: A. Systematic sampling B. Cluster sampling 4. A group of 50 teachers selected from 200 teachers represents the: A. Population B. Sample 5. Probability sampling is best used when: A. You want to generalize results to the population B. You are in a hurry Part III. Scenario-Based Questions Scenario 1 The Learner Council wants to know the favorite lunch meals of 900 learners. 1. Which method ensures fair results? A. Ask only council officers B. Ask friends during lunch C. Use random sampling D. Post only on social media 2. If they survey 10% of 900 learners, how many should be included? A. 45 B. 90 C. 100 D. 120 Scenario 2 A teacher surveys only one section and claims it represents all Grade 11 learners. 3. The result is an example of: A. Parameter B. Statistic C. Population D. Sample size 4. Why is this result not representative? A. It is too large B. Only one section was surveyed C. It used random sampling D. It included the whole population Scenario 3 5. A barangay health worker chooses: Option A: Random 100 learners Option B: 100 learners near the court Which is more representative? A. Option A B. Option B C. Both are equal D. Neither is valid
IV. Reflection:	*Share your feedback and reflect on what you have learned by answering the questions below.* 1. Why does random sampling give fairer results for the lunch meal survey than just asking friends or using social media? 2. Why is the teacher's finding a statistic rather than a parameter? Why is surveying just one section unreliable? 3. Why is Option A more representative than Option B? In what real-world health situation might a worker intentionally choose Option B? 4. Why is it important to calculate an exact sample size (like using Slovin's Formula) instead of just guessing a number?

	5. Choose one sampling method from the quiz. How would you use it to conduct a valid research study in your own school?
I. Activity 3:	**Extending and Reinforcing Learning**
II. Materials	pen and paper calculator
III. Instructions:	**For Remediation** **Part A. Sampling Method T-Chart** *Review the listed Probability and Non-Probability sampling methods in the T-Chart below. In the spaces provided, write* ***one (1) original, real-life school example*** *for each method.*

Probability Sampling	**Non-Probability Sampling**
Simple Random Sampling *School Example:*	**Convenience Sampling** *School Example:*
Stratified Sampling *School Example:*	**Purposive Sampling** *School Example:*
Systematic Sampling *School Example:*	**Snowball Sampling** *School Example:*
Cluster Sampling *School Example:*	

Part B. Sampling Match-Up

Match each situation with the correct sampling method from the box below. Write only the **letter** *of your answer in the blank provided before each number.*

A. Convenience Sampling
B. Stratified Sampling
C. Cluster Sampling
D. Simple Random Sampling
E. Systematic Sampling

1. A researcher surveys every 5th learner on the enrollment list.
2. A health worker randomly selects 50 learners from a master list.
3. A teacher chooses one section and surveys all learners in it.
4. A learner asks only his close friends for a quick survey.
5. A researcher divides learners by tracks (e.g., Academic, TechPro) and selects proportionately.

Part C. Real-Life Scenarios

Read each situation carefully. Show your solutions and write a brief explanation for your answers in the spaces provided.

Scenario One

A school has 1,200 learners. If the researcher wants to survey 10%, how many learners should be included in the sample?

- **Solution / Answer:**
- **Explanation:**

Scenario Two

In a population of 800 learners, 20% prefer online learning. If a sample of 200 learners is taken, how many are expected to prefer online learning?

- **Solution / Answer:**
- **Explanation:**

	Scenario Three A barangay has 10 blocks of houses. A researcher randomly chooses 3 blocks and surveys all households in them. • **What type of sampling is this? _____** • **Why is it classified under this method?** **For Enhancement** **Part A. Slovin's Formula Application** **Scenario:** The Mayor wants to survey the city (Population = 15,000). Use Slovin's Formula n=N1+Ne2 to calculate the required sample sizes based on different margins of error. Show your complete solution and give the final sample size. 1. Calculate for a **5%** margin of error **e=0.05** 2. Calculate for a **10%** margin of error **e=0.10** **Task:** Write a short policy memo addressed to the Mayor explaining why investing more money to survey the larger group (5% error) yields more reliable community statistics for the city. **Part D: Performance Task — School Wellness Program** **Scenario:** Your school is preparing a Wellness Program to support learners' mental and physical health. The goal is to make sure the survey represents the whole learner body fairly. **Application Questions:** 1. Who is your target population? 2. Which sampling method will you use? Why? 3. Suppose your school has 900 learners. How many will you include in your sample? (Show your choice of error margin and calculation) 4. What possible biases or problems might occur in your plan? 5. How will you avoid these problems to make sure your results are fair and useful? **Task:** Write a short report (5–7 sentences) summarizing your plan. Make sure to integrate your Introduction (purpose of the survey), Method (technique, sample size, justification), and Reflection (importance of fairness in sampling).
IV. Reflection:	*To help you reflect on your learning experience, answer the following feedback and reflection questions* 1. What is the difference between probability and non-probability sampling? 2. How do you decide which sampling method fits a given situation? 3. Why is sample size important in research? 4. What biases can happen in sampling, and how can they be avoided?